



Supporting more coherent and connected mental mathematics teaching

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Abstract

In this paper, we detail the outcomes of work in initial teacher education (ITE) that sought to support moves into coherent and connected early mental mathematics teaching in South Africa. Our aims in the paper are twofold. First, in revisiting an earlier framework (the Mediating Primary Mathematics framework), we elaborate a theoretically oriented and empirically sensitive continuum of teaching quality that moves from incoherence to coherence to connection to generality. Second, we present an analysis of the instruction of a cross-attainment sample of nine pre-service teachers from the ITE cohort at one university that indicates very few instances of the incoherence identified in earlier research, and substantial evidence of episodes that are, not only coherent, but also connected. The findings suggest that a combination of good quality structured teaching materials and course mediation of these materials can contribute to supporting good quality mental mathematics instruction.

Keywords Mental mathematics teaching · Coherence · Connections · Generality

Several studies over the last decade have provided evidence of problems with coherence and connection in mathematics instruction in South Africa (Adler & Sfard, 2017; Mathews, 2021; Venkat & Adler, 2012). One focus of earlier work undertaken within the *Wits Maths Connect-Primary* research and development project was therefore on developing frameworks for studying coherence in instruction and applying these frameworks to study classroom teaching. The Mediating Primary Mathematics (MPM) (Venkat & Askew, 2018) framework was developed during this working, with attention to formulating methodologies for exemplifying and evaluating differing extents of teaching geared towards the goal of bringing mathematical structure/generality into view (Askew et al., 2019). In that work, we described the strands of the MPM framework (tasks/examples elaborated in teaching through artifacts, inscriptions and different aspects of instructional talk) and illustrated different extents of teaching towards generality. We did this by analysing contrasts in episodes of instruction on different mathematical topics.

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In this paper, we turn to the underlying continuum of development which, in the Venkat and Askew (2018) paper, was simply stated as involving moves from incoherence to coherence to connection to structure and generality. This continuum is located within the idea of a “mathematical story” (Stein & Glenn, 1982). In mathematics, stories often begin in the context of an example that is presented or worked with. This story is then fleshed out via a set of examples. For instance, a story might begin with the example $5 + 5$, with 10 established as the sum. If the follow-up examples in the set are $6 + 4$, $7 + 3$, $8 + 2$ and $9 + 1$, there is potential for a story to emerge about the sum of ten being maintained under addition if one quantity is increased by an amount and the other quantity reduced by the same amount. Using Watson and Mason’s (2005) notion of “example spaces”, we view mathematical stories as beginning in learner engagements with the examples that are presented and worked with in the context of instruction. For *coherence*, each example has to be presented using artifacts/inscriptions that align well with the task it is set within, with instructional talk that involves mathematically legitimate offers or responses to learner thinking. If these conditions are not met because of poorly aligned representations and/or ambiguities and errors in teacher talk, we get *incoherence*. Mathematical stories begin within instruction that presents coherent examples. These stories expand as individual examples are linked in an example space connected to a task. A task and its associated example space comprise an “episode”. In some cases, a task within an episode may include, or be comprised of, sub-tasks each with an associated example space. Where episodes start to be *connected* in instruction, this enlarges the range of problems that students can work with. For instance, the example space in the sum of ten episode introduced above might be followed by an episode (with a task and associated example space) where students are asked to efficiently calculate the answer to questions like $99 + 101$ and $48 + 52$. When sequences of episodes are chained together with focus on the structure of a mathematical idea, we start to see powerful *generalised* understandings being made available to learn. An overview of the literature on coherence and connections, as key goals of mathematical learning, is presented later in the paper.

In this study, we observed nine pre-service teachers (PSTs) all teaching content linked to the “Jump Strategy”, a mental mathematics-based approach that uses number relations for working efficiently with two-digit addition/subtraction problems using number lines as the key underpinning model (Beishuizen, 1993). In this paper, we focus on two objectives: (1) we use the literature to elaborate the continuum from incoherence to generality and tailor it to the teaching of the Jump Strategy; and (2) we use the indicators developed for this elaborated continuum to analyse the extent to which PSTs, when supported to work with structured materials developed within the Mental Starters Assessment Project (MSAP) (Graven & Venkat, 2021) in their university B.Ed. course, were able to transcend the incoherence identified in earlier studies in their practicum teaching of the Jump Strategy.

In engaging with these dual objectives, we also address the concerns raised in Scheiner et al.’s (2024) paper highlighting the underrepresentation of studies from the Global South. This underrepresentation creates significant gaps in international understandings of the contexts, conditions and cultures of teaching and schooling in Global South settings.

1 The Mental Starters Assessment Project: motivation, model and materials

Problems with basic number sense amongst students (Graven et al., 2013) and amongst PSTs (Essien et al., 2023; Tshesane et al., 2025) have been extensively highlighted in South Africa. These problems build on ongoing evidence of children remaining stuck in counting in ones-based approaches for solving additive (Porteus & Mostert, 2022) and multiplicative problems (Schollar, 2008). Following interest from the South African Department of Basic Education in the development of formative assessments for use in the early primary grades, research teams linked with the South African Numeracy Chair projects collaborated with government personnel and other research and development partners to design teaching, learning and assessment materials that could support early number sense. The outcome was a set of mental mathematics materials, with associated assessments, for use within the 15-min “mental starter” segment of mathematics lessons at Grade 3 level (students aged 8–9) in South Africa.

The MSAP materials consist of six units, each focused on a particular mental mathematics strategy (Bridging Through Ten, Jump Strategy, Doubling and Halving, Rounding and Adjusting, Re-ordering, Relationship between Addition and Subtraction). Each unit begins with a 5-minute in-class pre-assessment that the teacher administers to her class, and then marks, to get a sense of initial strengths and weaknesses in the focal strategy. This is followed by eight starter activities that the teacher can use over the subsequent 3 weeks, concluding with a 5-minute in-class post-assessment. Marking the post-assessment provides the teacher with a picture of children’s learning of the strategy, and thus, a low-stakes feedback loop into her own teaching efficacy.

Using Askew’s (2012) framing of the “actions” of mathematical proficiency as based in fluencies, problem solving and reasoning, each unit includes tasks that range across these features. Within the MSAP materials, the terminology of “rapid recall”, “strategic calculating” and “strategic thinking” is used to refer to the specific ways in which our development of tasks links with the actions of mathematical proficiency:

- Tasks that assess awareness of underlying number relations needed at the level of rapid recall, or fluency, for the focal mental strategy to be brought into view
- Tasks that assess use of strategic calculating using the focal strategy in problem-solving tasks
- Tasks that assess reasoning about the structural equivalence relations that underpin the focal strategy through strategic (specifically focal strategy-related) thinking

In Fig. 1, each task type is exemplified drawing from the Jump Strategy unit pre-assessment.

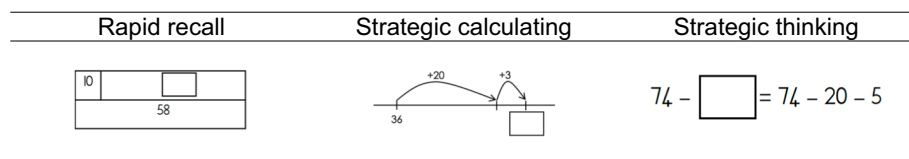


Fig. 1 Fluency, problem-solving and reasoning tasks from the Jump Strategy pre-assessment

The items also exemplify the inclusion of key representations (number lines and part-whole bar models) that have been noted as useful for supporting learning of the focal strategy. These representations feature extensively alongside symbolic number sentence representations across the units.

Each starter involves three “episodes”: a warm-up rapid recall episode, a whole-class teaching episode focused on strategic calculating and strategic thinking tasks that make use of the rapid recall skill and an independent practice episode with similar tasks. Tasks and their associated examples are suggested for use in each episode.

The MSAP materials were iterated across a sequence of research trials between 2017 and 2021 (see Graven & Venkat, 2021 and Askew, Graven & Venkat, 2022 for details of the promising outcomes that were seen across initial smaller scale and larger multi-province trials). In 2022, the materials were taken over by the South African Department of Basic Education and made available for national rollout on their website (<https://www.education.gov.za/MSAP2022.aspx>).

Given this availability for national rollout, it was important to bring initial teacher education (ITE) into the fray. In 2022, this led to the *Mental Maths-Work integrated Learning* (MM-WiL) project that brought together primary mathematics ITE lecturers to discuss how mental mathematics was incorporated into their programmes and whether the MSAP materials could be usefully integrated into their work.

In one urban institution’s ITE programme, Foundation Phase (Grades R–3) PSTs take mathematics modules across years 1 and 2 and methodology modules linked to these mathematics modules across years 3 and 4. In this programme, early number concepts including mental mathematics are dealt with in year 1. In 2022, 2 weeks of a third-year module were focused on discussing the MSAP model and materials and how the MSAP materials could be used in teaching. Arrangements were also made (where practicum schools gave permission for this) for PSTs to teach the Jump Strategy unit with a Grade 3 class during their 6-week practicum. In this cohort, PSTs’ submissions of pre- to post-test marks indicated gains aligned with those seen previously in the in-service trials (Venkat & Mathews, 2024). Given our prior attention to coherence and connection in instruction, this positive result led to interest in investigating the extent to which instruction at—at least the coherence level—might underpin any gains achieved. Lesson starters observation data from two Jump Strategy starters for a cross-attainment sample of PSTs in the 2023 cohort thus forms the dataset of interest in this paper.

2 Studying coherence and connections in instruction

Peters’ (2024) recent overview of studies focused on coherence notes two broad groupings: studies on curricular coherence and studies on instructional coherence. Our interest is in the latter. Internationally, a common thread in the latter body of work rests on connections. For example, in the TIMSS 1999 video study, Hiebert et al., (2003: 196) describe coherence as comprised by: “the (implicit and explicit) interrelation of all mathematical components of the lesson”. These interrelations construct, as we described earlier, a “mathematical story” within and across lessons (Stein & Glenn, 1982). Connections between examples within tasks, between tasks within episodes, between episodes within lessons and between lessons are the key mechanism through which mathematical stories are constructed, extended and expanded. Chen and Li (2010, p. 713) contend that this “story” is more than a sequence of events: “Each event must be organised and

interconnected such that the story has a beginning, middle, and end, as well as a consistent theme that runs throughout the lesson with a clear scheme". This view drew our attention to the need to consider the ways in which instruction draws attention to connections, between examples within an episode, and between episodes within and across starters (Cai et al., 2014). The incoherence, coherence, connection, generality continuum thus focuses on the nature and extent to which mathematical ideas are integrated and expanded within (Stigler & Perry, 1998) and across starters and lessons (Fernandez et al., 1992).

Peters (2024) also illustrates a range of approaches to studying coherence in instruction across these different temporal scales. South African studies on mathematics instruction have pointed to problems with coherence at the episode, or task-within-lesson level, with a range of methodologies and theoretical tools drawn on to study these disruptions to coherence. In Bansilal's (2012) study of a secondary teacher's instruction linked to a ratio task across several lessons, Sfard's (1992) theoretical tools are used to analyse mathematical errors and circularity within episodes, leading to what Bansilal describes as "a short-circuiting of the meanings and motives that underlie actions" (p.45). Similarly, the chapters in Adler and Sfard's (2017) volume study a break in coherence in a secondary teacher's instruction related to producing the solution to a quadratic inequality from a range of theoretical perspectives. Mathews (2021) uses a semiotic lens to explore examples of primary level division instruction in which sharing situations are solved with grouping actions, with no acknowledgement of the shift, nor a pathway through a chain of signifiers that could offer a rationale for the grouping actions. Venkat and Naidoo (2012) draw on systemic functional linguistics to illustrate what they describe as "extreme localisation", wherein examples in instruction are repeatedly handled in "stand-alone" rather than connected ways. The consequences of this, they argue, are limited openings for seeing some results as "established", and from which further outcomes can be derived.

This highlighting of problems at the example and episode levels led us into recent international studies that have explored the nature and range of links that can be made at the task/episode level in the context of classroom interaction. This literature offered descriptions of the kinds of links within examples that are necessary for coherent working and the links between examples and between episodes that build connections. Problematic links constitute disruptions to coherence. De Gamboa et al. (2023) offer a useful classification of links in the following terms:

- Representational links (pointing out equivalent/alternate representations)
- Common feature links (common features of definitions and representations or metaphorical projections)
- Procedural links (linking a concept and procedure or equivalent procedures)
- Argumentative links (including justifications, if-then arguments and particularisation/generalisation)

This classification was useful for studying the kinds of connections that PSTs made in their teaching of lesson starters and also for studying instances of "contraindicators" of connections where a connection was disrupted in some way. This was important in a context where incoherent assemblies of representations had been reported in teachers' presentations of mathematical ideas. In another ITE project where students were introduced to the MSAP materials, Lovemore and Graven (2024) offer evidence of improvements in PSTs' *university-based microteaching* of the Jump Strategy, seen in their use of the empty number line and accompanying talk and gestures. Our interest was in exploring whether ITE working with the MSAP materials could support moves towards coherence/connection in instruction in PSTs' *school-based practicum teaching*.

3 A continuum towards generality: Theory and indicators

As noted already, the earlier MPM model simply stated a trajectory that began with incoherence at the lowest level, coherence at the second level, connection at the third level and generality as the fourth level. We use the term “continuum” here to note that this is not a “ladder” where all teachers start at the bottom rung. Instead, instructional practice involves moves in multiple directions in response to task demands and learner understandings. Further, it is important to note that the continuum does not represent a linear sequence within instruction. By way of example, a teacher might begin a starter in the Jump Strategy sequence with a reminder to “always jump in tens and ones.” This is a statement with a level of generality, but if she sees a student then proceeding to count on in ones, responsive instruction may require some returning to recording place value decomposition of two-digit numbers and practising forwards and backwards jumping in tens on a number line and/or using a 1–100 number chart. This “step back” would require coherent working with examples and connecting between examples to establish fluency with the more basic skills of decomposition and counting on and back in tens from any number. These fluencies are necessary for using place value decomposition of addend/subtrahend for efficient calculation with the Jump Strategy. There are also, other, interim generalities in this sequence—for example, knowing that when jumping forward or backward in tens in the decimal number system, the units digit remains unchanged (e.g., 73, 63, 53, 43, etc.).

Instruction, therefore, may well require backward and forward movements along the continuum, and possibly, the implied, rather than overt, presence of an intermediate level. Across this traversing though, goals relating to structure and generality remain critical for supporting the establishment of ideas that are important for building further learning experiences on. Consequently, instructional practice can be examined in terms of the extent to which evidence of attention to structure and generality is present, with the continuum serving as a heuristic for recognising strengths, identifying opportunities for growth and supporting the enhancement of such practices over time.

Venkat and Askew (2018) detail the sociocultural bases of development of the MPM model, with emphasis on tasks/example spaces mediated through the assembly of manipulatives, representations and instructional talk to build: “a press towards generality and/or domain-specific ways of thinking” (p.73).

Drawing from the literature overviewed above, and the variation theory-based writing on example spaces (Marton & Booth, 1997), we were able to elaborate each level of the continuum from incoherence to generality. This provided an outcome related to our first objective: delineating the continuum.

The base level of *incoherence* reflects phenomena reported in the South African ground. While mathematical errors are highlighted in international frameworks for studying the mathematical quality of instruction (Hill et al., 2008), incorrect answers were relatively rare in our early grade mathematics classroom observations: incoherence was manifested in other ways. Mathews (2021) presents analysis of a teacher explaining how to work out the answer to sharing 13 sweets between 3 children. The situation is linked to a drawing of groups of 3 sweets. The teacher counts in multiples of 3 to 12 across four groups and draws an additional dot outside of the groups to produce 4 remainder 1 as the answer. The disconnect between “3” representing the number of children in the task and the number of sweets given to each child in the explanation is not addressed. Bowie et al. (2025) note instances where teachers, dealing with problems like $__ - 38 = 52$, calculate the answer of 90 for

themselves by adding the two given numbers and say they would explain how to work out the answer using a number line showing 90–38. Here, the answer problematically appears at the start of the instructional explanation.

Coherence involves mathematically legitimate offers of, or responses to, problem-solving processes for particular examples. Representations and instructional talk need to align well with the strategic aspects in focus. Fundamentally, a coherence coding involves examples worked with in legitimate ways. These ways may include working that involves inefficient approaches, such as presenting or accepting counting-in-ones approaches to solving early number problems.

Connecting between examples which are each handled coherently provides the key indicator for a “connection” coding. This coding is responsive to the “extreme localisation” identified in Venkat and Naidoo’s (2012) analysis, emanating from evidence of examples repeatedly handled with counting in ones, with no reference to previously established results. *Connected working* is predicated on bringing examples (and later, episodes and learning across episodes) into relation with each other. For example, calculating $22 - 5$ as $22 - 2 - 3$ involves connecting the more complex, initial calculation with several more basic fluencies, in this case, knowing the multiple of ten before 22, the relationship between 22 and 20, and the decomposition of 5 needed to bridge through 20 as an efficient calculation strategy. Connections therefore expand the example spaces that students can navigate independently, through instruction that provides/supports access to the development of the required mathematical proficiencies. In the MSAP materials, the use of skills developed in the rapid recall episode within the strategic calculating/strategic thinking tasks episode and the independent working tasks episode in each unit offered a structure that, in intention, was predicated on connections.

Coherence is a prerequisite for connections and connections are a prerequisite for generality. Navigating example spaces rather than individual examples demands an understanding of how tasks, examples and episodes are connected. In mathematics, example spaces can be connected by a procedure, or a representation, that can be applied across a set of examples. Discursively, instruction that builds these connections will include reference to what varies and what remains invariant across a set of examples. Askew (2019) illustrates instruction that exhibits strong attention to connections in a lesson focused on place value, seen in “frequent and explicit moves between working with and between composite and single units” (p. 224). He notes that working in this way encouraged students in the class to actively focus on connections and generalisations.

Generality is marked by offers, or responses, that apply across a domain. This can be a general form of a particular procedure—for example, recurrently using place value decomposition of the addend or subtrahend in problems to calculate the answer to addition or subtraction problems efficiently, with awareness of the approach’s general applicability (e.g., $45 + 23$ calculated as $45 + 20 + 3$ or $72 - 35$ calculated as $72 - 30 - 5$). Generalised talk is presented across several aspects of the MSAP Jump Strategy starter materials, for example, in the repeated noting that adding involves starting at the left hand end of a number line to jump forwards, while subtracting means starting at the right hand end to jump backwards.

This elaboration of levels of the continuum towards generality using the literature base on additiverelations teaching and the MSAP Jump Strategy unit materials addressed our first objective with some fleshing out of indicators for each level. Given that instructional working occurs through both teaching and eliciting/responding, we formulated indicators for both of these routes. In Table 1, we detail general indicators and also include instances of these indicators tailored to the Jump Strategy (these are italicised in the table). Our empirical data

pointed to the need to include what we describe in Table 1 as “contraindicators” for some levels in our model. These were phenomena that suggested shortcomings with consistent maintenance of the descriptive bar for a level within an episode.

In earlier MPM working, responsive teaching was seen as a marker that only came into view at the connected level. Our inclusion of responsive teaching (eliciting/responding) at all levels was driven by instances where we could differentiate lower level “corrective” responses from more elaborated “rationalising” responses. In the latter, PSTs offered approaches or questions for

Table 1 Continuum towards generality: levels and indicators

Incoherence	Teaching	*Instructional talk, representations and tasks do not align. Enactment is not mathematically legitimate
	Eliciting/ responding	Teacher elicits/responds to student inputs in ways that are not mathematically legitimate
Coherence	Teaching	Instructional talk and representations align well with task/examples and are mathematically legitimate Contraindicator: Limitations/shortcomings in alignments between task, talk and representations
	Eliciting/ responding	Student offers are accepted where appropriate/corrected when inappropriate. Correction comes from a teacher offer or from another student <i>**Correcting non-standard expressions and/or non-standard number line representations</i>
Connection	Teaching	Connecting across coherent examples/episodes <i>Modelling fluency with counting on in tens/multiples of ten within teaching of jump strategy</i> Contraindicator: Demonstrating inefficient approaches <i>Including counting on in ones within instruction</i> Contraindicator: Demonstrating different approaches for same skill across tasks/representations with no rationale <i>Offering different methods for adding ten across episodes</i>
	Eliciting/ responding	Noticing differences between strategies <i>E.g., between split and jump strategies, and guiding towards jump strategies for mental working</i> Eliciting/responding in ways that offer rationales for how to produce correct answers or to advance thinking through instructional explanation or through guided questioning that supports advances in student competences Contraindicator: No response to inefficient strategy use <i>No noticing of students counting on in ones</i> Contraindicator: Rigid focus on a singular efficient strategy <i>Teacher rigidly sticks to “not counting in ones” even when student appears unable to use recalled facts</i>
Generalisation	Teaching	Generalised talk relating to concepts and procedures <i>Messaging addition as forward moves/subtraction as backward moves on number line</i> Generalised representation <i>Symbolising place value decomposition as general underlying structure of jump strategy</i>
	Eliciting/ responding	Eliciting/responding to student inputs on general structure of jump strategy

*Text in normal font refers to general specifications

** Text in italics refers to Jump Strategy-related specifications

producing correct answers in ways that advanced students' competence for independent working. The former sits within the coherence stage, while the latter fits within the connections stage.

This elaboration of the continuum levels with indicators made it possible for us to turn to our dataset for qualitative analysis of episodes of teaching. We were particularly interested in whether discussion and use of the structured MSAP teaching materials in ITE could help to address the concerns with incoherence in instruction reported in previous studies.

4 Methodological approach and data sources

Favourable pre- to post-test gains based on PSTs' practicum teaching of the Jump Strategy in 2022 led to interest in exploring the prevalence of incoherence/coherence/connection/general-ity in PST instruction. Observations of starters taught by PSTs were added into the 2023 ITE course iteration with these observations included in the research dataset (these had not been part of the 2022 pilot). The mean pre- to post-test gain based on practicum Jump Strategy teaching was maintained in the 2023 cohort: among the 58 out of 70 PSTs that were able to complete the teaching of the unit and submit pre- and post-assessment marks for the Grade 3 class they taught, a mean gain of 16.1 percentage points was seen based on $n=1275$ matched Grade 3 students.

Given evidence of association between mathematical content knowledge for teaching and the mathematical quality of instruction (Hill et al., 2008), we wanted to observe a cross-attainment sample of PSTs' instructional practice. Prior mathematical attainment was gauged by combining school-leaving equated Mathematics/Mathematical Literacy scores (Bridgman, 2020) and first- and second-year B.Ed. Mathematics course marks. Ranked overall scores were then divided into three attainment bands to provide rough categories across the attainment range for sampling from the overall B.Ed. third-year cohort. Twenty PSTs, ranging across these attainment bands, were initially approached to be part of the observation sample, with the exploratory and non-evaluative focus of our videorecording of two of their lesson starters stressed. Thirteen PSTs gave informed consent, but we were able to collate full video data from two lesson starters for nine of these PSTs. Two of these PSTs were classified as mathematically low attaining (T1–2), four were in the mid-attaining category (T3–6) and three were high-attaining (T7–9) based on our initial three broad attainment bands. We arranged observers to video-record the two starters, with the video capturing the PST's instructional talk, inscriptions on the board, any artifacts used alongside, and their responses to children's offers and questions. Ethical permissions for our data gathering were sought and received from the school principal, each PST, children in the class and their parents/carers following university approval for the study (clearance number H21/11/45).

Our video dataset comprised the two lesson starters taught by each of the nine PSTs—approximately 30 min of video data in total for each PST. As noted earlier, the MSAP lesson starters as presented in the MSAP materials contained three episodes: a rapid recall task segment, a strategic calculating and thinking task segment and an independent practice task segment. However, some PSTs added other episodes or did not include one of these episodes. Enacted episodes (based on PSTs' demarcated task segments) were included in our analysis. Across the nine PSTs' 18 lesson starters, there were a total of 56 episodes.

Our process of analysing the video data consisted of the following: (1) independently watching the same three PSTs' videos—one from each of the attainment categories described above—to give us access, theoretically, to maximal variation in instructional quality. Using the indicators/contraindicators summarised in Table 1, we identified instances of incoherence, coherence, connection and generality in the starter; (2) coming together to share our identifications, discuss any differences and reasons for these, and clarify/adjust our understandings of the level indicators; (3)

following these clarifications, each of us going on to complete the analysis of video data for three PSTs, creating summary tables of the tasks and examples set within each episode, and narratives of the overall starter (i.e., across all episodes) on the spread of levels that could be inferred. An exemplar summary table for T3's first observed starter is included in Table 2.

The Table 2 summary allowed us to note that T3's starter teaching was primarily located at the level of connection. The presence of a contraindicator to connection pointed to the

Table 2 T3 Starter 1 summary

Content broadly aligned with Jump Strategy Starter 1, but no direct reference to tasks/examples suggested in the MSAP materials

Episode 1	Episode 2	Episode 3	Episode 4
Warm-up: Counting forward by tens 10 to 100 Counting backward by tens 100 to 10 Counting forward by tens 5–105 Counting backward by tens 105–5 Counting forward by tens 9–109 T3 corrects when incorrect answers are heard in class choral responses	+ 10 written number sentences: $46 + 10 =$ $78 + 10 =$ $68 - 10 =$ Across all, T3's solution process rests on adding/sub- tracting 1 from the tens digit and moving across the units digit: (e.g., for $46 + 10$, T3 asks class to add 1 to the 4 in 46 and writes 5. Then "moves across" the 6 in 46, to get 56 as the answer). Emphasises not counting on by ones	Main Jump Strategy activity: $79 + 11$ Worked through as in MSAP. Students call out + 10 and + 1 decomposition of 11 immediately. T3 uses conventional recording on number line. Two versions of number sentence written below: $79 + 10 + 1 = 90$ $79 + 11 = 90$	Individual Jump Strategy work: $54 + 13 =$ Student called up to show working on board. T3 corrects to conventional number line representation when student writes 54 above the number line Reminds class to avoid counting in ones. Student at board adds on 10 and 3 men- tally and records answers on number line T3 writes $54 + 10 + 3 = 67$ and $54 + 13 = 67$ One student seen counting on 3 on her fingers. T3 response not seen on video $64 - 12 = \underline{\quad}$ set for further independent work

Incoherence/error:

- None

Coherence

- All tasks across all episodes are dealt with coherently. All talk and inscriptions linked with procedures and their associated representations are mathematically justifiable

Connection

- Across all episodes, the need to jump in tens when adding or subtracting is contrasted with counting in ones, with jumping in tens advocated. Jump strategy is linked with "counting in tens" (Episode 1), with class reminded in Episode 2 not to count in ones when calculating $46 + 10$. Counting in tens is then used in Episodes 3 & 4 jump strategy tasks, with T3 reminding class again in Episode 4 to avoid counting in 1s
- Contraindicator: The "method" for adding tens differs across oral statements of adding ten within examples and direct written number sentences involving adding 10. In the oral counting in tens in Episode 1, the class are expected to know the next ten/tens before in their forward/backward counts. In Episode 2, a different digit-wise procedure is offered for writing the answer when ten is added, with no reference back to Episode 1 counting forward/backward in tens. In Episode 3, there is reversion to Episode 1 fluency-based stating of answer when adding/subtracting ten in jump strategy calculating. This is carried through into Episode 4
- Connected also to student responses—alert to errors in responses and representations; steps in with support or corrections on both. Remediation seen here and extension in advocacy of moving beyond counting on in ones. Some students able to mentally add the tens and units, but one student in independent working seen adding the units figure by counting on 3 on her fingers

Generalisation

- Working with addition and subtraction using counting in tens, through connection across all four episodes, starts to function as a generalised approach to use, in spite of the disruption to this generality in the different procedure for adding ten in Episode 2

Overall

No incidences of error/incoherence; all teaching falls within the bounds of coherence; some evidence of connection, though this is disrupted in the lack of pulling through of fluency with adding/subtracting ten into Episode 2. However, the notion of using counting in tens when doing addition/subtraction features as a generalised approach

need for support at this level, but there were also some indications of T3 starting to see adding tens as a generalised approach that could be used across multiple tasks and examples. Given the prevalence in South Africa of inefficient counting-based approaches, such evidence linked to the Jump Strategy approach presented in the MSAP materials and discussed in course teaching, suggested promise for more connected instruction.

5 Analysis

Our analysis proceeded using the approach outlined above, with similar qualitative summaries produced for the 18 lesson starters taught by the nine cross-attainment PSTs in our sample. This allows us to understand the extent to which instruction in the context of PSTs' work with the MSAP materials transcended incoherence on the one hand and involved instruction that could be interpreted as coherent/connected/generalised on the other hand.

While our coding for incoherence/coherence/connection/generality was focused on episodes, our overall commentary is focused at the level of the 18 lesson starters. This is due to the fact that the mathematical "story" linked to jump strategies instruction emerges at the level of the lesson starter, with connections (or not) between examples and across episodes within starters constituting a key part of this story.

5.1 Incoherence

Across the 56 episodes observed in the 18 starters in our sample, three episodes—all drawn from T1—indicated evidence of some incoherence in instruction. In two instances, incoherence was linked to this teacher's instructional talk and inscriptions while presenting addition and subtraction using the Jump Strategy. In T1's first starter, while working with tasks $23 + 14$ and $45 + 23$, we noticed that after decomposing the added number correctly, he inscribed the forward jumps to be made as " $10 +$ " and " $4 +$ " instead of " $+ 10$ " and " $+ 4$ " in the first task, and similarly as " $20 +$ " and " $3 +$ " in the second task. T1 expressed the additions to be carried out, in two instances, following these inscriptions: the answer to "four plus thirty-three" was asked for within the first task and the answer to "twenty plus forty-five" was requested in the second task. Students produced the correct answers to these questions and to the two overall problems in both cases, but it is worth noting that the number line representation for $45 + 20$ would differ from the representation for $20 + 45$ (see Fig. 2). T1's asking for the answer to $20 + 45$ therefore did not cohere with his number line diagram which showed $45 + 20$.

In the second starter, T1—while demonstrating how to answer $78 - 24$, interchangeably used "seventy-eight minus twenty" and "twenty minus seventy-eight" in his instructional talk. These two versions, of course, do not give the same answer. In both lessons, it is worth noting that the class offered the correct answer, which was then recorded on the number line. It is also worth noting that the individual working seen on the video showed children following T1's idiosyncratic recording with correct answers. This suggests that the issue is not one of conceptual understanding of the Jump Strategy on either T1 or the students' parts, but rather gaps in appropriation of the correct expression and standard inscriptions linked with the jump strategy.

The third instance of incoherence was seen in T1's fluency activity in the first starter. Setting the task "What is the next multiple of ten?"—he proceeded to accept student answers that reflected "ten more" than his set of examples. This suggests a lack of

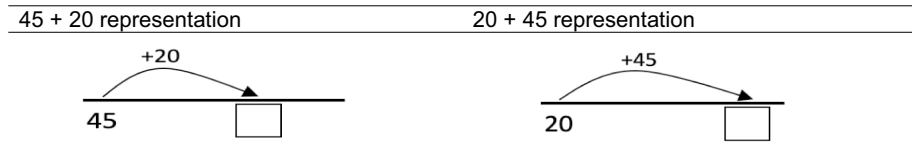


Fig. 2 $45 + 20$ and $20 + 45$ represented on number lines

awareness of the distinction between “next multiple of ten” and “ten more” than a given number, with addressing of this confusion featuring within subsequent course iterations (Prinsloo & Mathews, 2026).

All three cases of incoherence relate to clarity of mathematical expression rather than conceptual misunderstanding on the teacher’s part. T1 was the teacher with the weakest content knowledge profile in our sample. Overall, the low incidence of incoherence was a positive outcome in the light of the previous South African evidence of incoherence in instruction (Venkat & Naidoo, 2012).

The remaining 53/56 episodes involved work with talk and inscriptions related to procedures and their associated representations that we interpreted as mathematically legitimate, and therefore, at least at the level of coherence.

5.2 Coherence

Several instances showed PSTs’ alertness to coherent working, with their correcting of errors in student responses and editing of number line representations into “standard” formats. In six episodes we noted contraindicators to coherence, located within T1, T2, T4 and T5’s instruction. In T1’s second starter teaching, his fluency task asked for the multiple of ten before a series of numbers. His series involved several multiples of 10 and 25 as the last example. Across the multiples of ten examples, the class offered the previous multiple of ten. For 25, 15 was offered. T1 corrected this answer, re-stating that he wanted the “multiple of ten before” and noting that was 20. This episode is not incoherent, but T1’s examples meant that “ten less than” and the “multiple of ten before” a given number coincided. The last example surfaced this misconception, and T1’s corrective response showed a responsive connection with learning, but most examples missed the error.

While showing students the adding 6 step in the Jump Strategy teaching of $35 + 16$, T2, in recording the jumps of 5 and 1, represented 1 with a larger jump compared to the jump of 5. Correct answers were elicited and written in, but there were instructional shortcomings in the representation produced from a “supporting number sense” perspective. T4, across both the middle Jump Strategy teaching episodes in her two starters, repeatedly used the phrase “count on” instead of “count back from” for subtraction. For $57 - 24$, after subtracting 20, she asked students to “count on” from 37 to arrive at 33. Additionally, in one instance, T2 simply ignored an incorrect answer of 78 for $47 + 34$ and moved on to another task. Similarly problematic for responsive instruction, T5 offered no alternatives to a student who was unable to produce the answer to a jump of 5 from 34 beyond cautioning the student to not count with fingers and eventually recording the correct answer from other students. In this last case, the student was given access to the correct answer, but no input on how to produce it.

The small number of contraindicators in this category suggested two outcomes. First, the structured support offered through lectures on the MSAP materials appeared to have allowed the vast majority of instruction seen to surpass the bar for coherence. Second, further support on precise use of mathematical language (across home languages and English), ensuring selections of examples that cohere well with the task, and on pathways to efficient strategies appears necessary for strengthening coherence for some lower/mid-attaining PSTs.

5.3 Connection

The vast majority of episodes in our dataset were located in the connection category. Given earlier evidence of incoherence and poor connection, this was a positive outcome. Given the extent of data in this category, there are more sub-themes and contraindicators in this section.

Across all 9 PSTs and the 18 lesson starters in the dataset, a universally observed connecting idea across episodes was reference to counting in tens rather than in ones. This recurring reference was important given the widely documented use of unit counting (Porteus & Mostert, 2022). In the working seen in our video data, most students were able to produce the answer to ten more/ten less than a given number through rapid recall across both the fluency, Jump Strategy and independent practice episodes. Only two instances of children counting the whole addend/subtrahend in ones were overtly visible in the whole-class teaching episodes in both of T2 and T4's lesson starters.

Thirteen of 18 lesson starters included examples that involved addends/subtrahends with multiples of ten (e.g., $39 + 23$). Given interest in progression towards more efficient calculation, we explored teachers' comparison of multiple jumps of 10 with a single jump of 20/30. In T2's Starter 1, we saw no attempt to mediate student working with such examples. T6, in her Starter 2, also only used jumps of 10. In five starters, teachers presented jumps of 10 vs. jumps of multiples of 10 simply as alternatives. In six starters, jumps of 20/30 were either offered or accepted. Only two teachers (T6 Starter 1 and T7 Starter 2) made reference to a single longer jump of a multiple of ten being more efficient than multiple jumps of ten. In both cases, this reference was oblique, rather than direct. T7, while working on $47 + 21$, said she would go with one jump of 20 because the class were "all very smart". T6 acknowledged two jumps of ten as "easier" than a single jump of 20. The latter point was interesting given that T6's fluency activity: counting on and back in twenties from particular start numbers, proceeded smoothly suggesting that advance into a single jump of 20 was within reach for the class.

Efficient work with the Jump Strategy requires fluent counting forward/back in tens and adding/subtracting single digit numbers to any number. We noted three overt instances across the 56 episodes of children using their fingers to count on or back the ones part of the addend/subtrahend. T4 demonstrated the units jump using counting on/back in ones. The evidence suggests that across both the MSAP materials and in teacher education, PSTs require guidance to support and advocate for adding/subtracting a single digit to any number to be seen as a fluency, rather than being enacted through counting.

In all except T1 and T3's lesson starters, episodes were well-connected. We have already noted the instances in T1's case of incoherence which limited the potential for connection, in spite of generally coherent understanding of the structure of the Jump Strategy for the teacher and the students. In T3's case, while the Jump Strategy episodes were taught coherently, an additional episode was inserted between the initial warm-up episode and the

whole-class Jump Strategy teaching episode in both her starters. In this additional episode, focus was on written number sentences involving adding ten (first starter) and subtracting ten (second starter), for example, $46 + 10 = \underline{\quad}$ and $63 - 10 = \underline{\quad}$. In spite of the class's fluency with counting forwards/backwards in tens, T2 presented an alternative procedure in her additional episodes for writing the answer: adding/subtracting 1 from the tens digit (e.g., the 4 in $46 + 10$ becomes 5) and gesturing a move across of the units digit (i.e., the 6 moves across to give 56). In this working, there is a focus on "digits", which breaks with the Jump Strategy focus on the "quantities" represented by both numbers in the calculation. While her procedure was mathematically legitimate, these additional episodes interrupted, rather than supported the "numerical quantities" story that is central to Jump Strategy working. Thus, they were coded as illustrating contraindicators of connection.

Contrasting with these interruptive instances, two PSTs responded sensitively to student responses that used traditional digit-wise decompositions. T9 intervened gently to a student that started $43 + 24$ with $40 + 20$, asking him to try approaching the problem by splitting only the 2nd number. The student's strategy was not disputed; rather, T9 expressed awareness that the Jump Strategy worked with a different decomposition of the numbers. Similarly, T2 showed responsive remediation in her Starter 2 teaching of what split of 6 to use in the question $35 + 16$ after the first split of 10 and 6. Responding to confusion, she asked for all possible splits of 6 and then asked: "At 45, how many jumps does she need to get to 50?"

The extensive presence of instances of connection was a particularly positive result, given the mathematics education literature base's agreement that connections underlie both conceptual understanding and the production of a mathematical story.

5.4 Generality

Two aspects of generality linked to jump strategies were seen in all but 1/18 lesson starters. First, all PSTs, with the exception of T1 in Starter 1, communicated addition as being represented by forward jumps and subtraction by backward jumps on the number line, highlighting what De Gamboa et al. (2023) describe as common feature links. Second, all nine teachers either offered, or accepted, student inputs that exemplified the general structure of the Jump Strategy model. In T1's case, a student or the teacher orally stated the separate jumps they made, for example, for $68 - 14$, saying "Sixty-eight minus ten equals fifty-eight" and "fifty-eight minus four equals fifty-four". In all other cases, some written record of the Jump Strategy structure was incorporated. Most minimally, this involved a single number sentence such as $68 + 30 + 5 = 103$ following the number line jumps for $68 + 35$ (T6). Three teachers (T3, T8 and T9) consistently recorded two or more equivalent statements. T8 incorporated attention to bridging through ten within her Jump Strategy tasks and recorded this structure as follows:

$$25 + 18 = 43$$

$$25 + 10 + 8 = 43$$

$$25 + 10 + 5 + 3 = 43$$

T5 represented the Jump Strategy structure using equivalent expressions such as $64 + 12 = 64 + 10 + 2$, the only teacher to use this format.

Overall, the data pointed to wide awareness of the structure of the Jump Strategy as a useful general approach for efficient mental calculation in the context of 2-digit addition and subtraction.

6 Findings and conclusions

Our analysis confirmed the result we were seeking: a move beyond instances of incoherence across almost all cases. T1 was the only teacher with any episodes falling into this category, and the number of episodes classified as containing incoherent elements was small (3/56). Instances of incoherence were linked to lack of clarity in number line representations and the language in working with these, rather than to the underlying structure of the Jump Strategy procedure. Thus, even amidst concerns about incoherence, T1 and his students were able to use Jump Strategy approaches correctly to answer problems. However, tailored feedback to support representational clarity is needed here to ensure full coherence.

More substantively, we have extensive evidence of the threshold for coherence being surpassed. This suggests that we can turn our focus in teacher education predominantly to the connections level using our indicators and contraindicators. The contraindicators, in particular, offer illustrations for discussion in our future teaching of mental mathematics, for example, on how to deal with students answering questions like $43 + 24$ by starting with $40 + 20$, instead of the $43 + 20$ jump advocated within the Jump Strategy. By extending this discussion to include examples like $61 - 24$, in which splitting both numbers surfaces common difficulties for children, we can highlight the advantages of using jump strategies. Such examples offer a more advanced “horizon” for teacher knowledge (Ball & Bass, 2009) than we began the MM-WiL project with. The MSAP materials, as presented and used in this B.Ed. programme, do appear to have facilitated mental mathematics instruction for most of our PSTs in ways that shift our attention to connections and generalisations rather than firefighting for basic coherence. This is an important shift in base level and points to the usefulness of combining good quality structured teaching materials with supportive teacher education for facilitating instructional enactments that are productive for learning.

The data also point us to gaps in student knowledge of some basic skills that we would, ideally, like to see at the level of rapid recall (for example, adding or subtracting a single digit number to/from a given number) and limited responsive PST emphasis on supporting the appropriation of this fundamental knowledge. In future iterations, we are likely to need to mediate the use of the MSAP materials in ways that include attention to other basic skills beyond using counting in tens rather than ongoing counting in ones.

Learning outcomes data and studies of early mathematics instruction in other low-attainment contexts suggest that a continuum that focuses attention on connection and generality in instruction is of broader interest. This is likely because the continuum is attuned to fundamental aspects of how mathematical understandings are built in instructional settings, via examples and example spaces. In this, there are also theoretical and methodological contributions. An approach based on the idea of expanding the example spaces that students can operate within confidently and efficiently (Watson & Mason, 2005) underpins the trajectory towards generality. In this approach, students are taught how to work with particular examples, to then see these examples as a set connected by a particular fluency, before being introduced to examples where this fluency/combination of fluencies is applied.

Representations are key to making the generalised structure visible. Through this approach, we have sharpened the descriptions of the underpinning trajectory. Additionally, the differentiating of lower and higher level responsive feedback in instruction has been highlighted.

The idea of expanding example spaces and findings on transcending incoherence will be incorporated into future iterations of the B.Ed. course as part of post-observation feedback. While our working with the four levels in the context of Jump Strategy instruction was exploratory and qualitative, the indicators and contraindicators developed in this analysis bring possibilities for future larger-scale episode-based quantitative evaluations of teaching quality in mathematics into view.

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Data availability The informed consents provided by the study participants as part of the University-approved ethical permissions in this project (clearance number H21/11/45) allowed for research team access to the dataset only, with no permissions for broader data sharing.

Declarations

Conflict of interest Anthony A. Essien is an Associate Editor of the journal. Rigorous peer review was not compromised as the journal has systems in place to ensure that peer review is protected from conflicts of interest.

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