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Dyscalculia and Mathematical Difficulties: Implications for Transition to Higher Education in the Republic of Ireland

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Abstract

This paper examines the neurological, cognitive and environmental features of dyscalculia, which is a specific learning difficulty in the area of processing numerical concepts. A review of the literature around the aetiology of dyscalculia, methods for assessment and diagnosis, global incidence of this condition and prevalence and type of intervention programmes is included.

In addition, the nature of dyscalculia is investigated within the Irish context, with respect to:

- the structure of the Mathematics curriculum
- access to learning support
- equality of access to the Mathematics curriculum
- reasonable accommodations and state examinations
- implications for transition to higher education

Finally, provision of Mathematics support in third level institutions is discussed in order to highlight aspects of best practice which might usefully be applied to other educational contexts.

Section 1: Literature review

1.1 Introduction

Mathematical skills are fundamental to independent living in a numerate society, affecting educational opportunities, employment opportunities and thus socio-economic status. An understanding of how concepts of numeracy develop, and the manifestation of difficulties in the acquisition of such concepts and skills, is imperative. The term Dyscalculia is derived from the Greek root 'dys' (difficulty) and Latin 'calculia' from the root word calculus - a small stone or pebble used for calculation. Essentially it describes a difficulty with numbers which can be a developmental cognitive condition, or an acquired difficulty as a result of brain injury.

Dyscalculia is a specific learning difficulty that has also been referred to as 'number blindness', in much the same way as dyslexia was once described as 'word blindness'. According to Butterworth (2003) a range of descriptive terms have been used, such as 'developmental dyscalculia', 'mathematical disability' , 'arithmetic learning disability', 'number fact disorder' and 'psychological difficulties in Mathematics'.

The Diagnostic and Statistical Manual of Mental Disorders, fourth edition (DSM-IV) and the International Classification of Diseases (ICD) describe the diagnostic criteria for difficulty with Mathematics as follows:

> **DSM-IV 315.1**
> **'Mathematics Disorder'**
>
> Students with a Mathematics disorder have problems with their math skills. Their math skills are significantly below normal considering the student's age, intelligence, and education.
>
> As measured by a standardized test that is given individually, the person's mathematical ability is substantially less than you would expect considering age, intelligence and education. This deficiency materially impedes academic achievement or daily living. If there is also a sensory defect, the Mathematics

deficiency is worse than you would expect with it. Associated Features:

Conduct disorder
Attention deficit disorder
Depression
Other Learning Disorders

Differential Diagnosis: Some disorders have similar or even the same symptoms. The clinician, therefore, in his/her diagnostic attempt, has to differentiate against the following disorders which need to be ruled out to establish a precise diagnosis.

WHO ICD 10 F81.2
‘Specific disorder of arithmetical skills’

Involves a specific impairment in arithmetical skills that is not solely explicable on the basis of general mental retardation or of inadequate schooling. The deficit concerns mastery of basic computational skills of addition, subtraction, multiplication, and division rather than of the more abstract mathematical skills involved in algebra, trigonometry, geometry, or calculus.

However it could be argued that the breadth of such a definition does not account for differences in exposure to inadequate teaching methods and / or disruptions in education as a consequence of changes in school, quality of educational provision by geographical area, school attendance or continuity of teaching staff. A more helpful definition is given by the Department for Education and Skills (DfES, 2001):

> ‘A condition that affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers, and have problems learning number facts and procedures. Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence.’

Blackburn (2003) provides an intensely personal and detailed description of the dyscalculic experience, beginning her article:

> “For as long as I can remember, numbers have not been my friend. Words are easy as there can be only so many permutations of letters to make sense. Words do not suddenly divide, fractionalise, have remainders or turn into complete gibberish because if they do, they are gibberish. Even treating numbers like words doesn’t work because they make even less sense. Of course numbers have sequences and patterns but I can’t see them. Numbers are slippery.”

Public understanding and acknowledgement of dyscalculia arguably is at a level that is somewhat similar to views on dyslexia 20 years ago. Therefore the difference between being ‘not good at Mathematics’ or ‘Mathematics anxiety’ and having a pervasive and lifelong difficulty with all aspects of numeracy, needs to be more widely discussed. The term specific learning difficulties describes a spectrum of ‘disorders’, of which dyscalculia is only one. It is generally accepted that there is a significant overlap between developmental disorders, with multiple difficulties being the rule rather than the exception.

1.2 Aetiology

According to Shalev (2004):

> “Developmental dyscalculia is a specific learning disability affecting the normal acquisition of arithmetic skills. Genetic, neurobiologic, and epidemiologic evidence indicates that dyscalculia, like other learning disabilities, is a brain-based disorder. However, poor teaching and environmental deprivation have also been implicated in its etiology. Because the neural network of both hemispheres comprises the substrate of normal arithmetic skills, dyscalculia can result from dysfunction of either hemisphere, although the left parietotemporal area is of particular significance. Dyscalculia can occur as a consequence of prematurity and low birth weight and is frequently encountered in a variety of neurologic disorders, such as attention-

deficit hyperactivity disorder (ADHD), developmental language disorder, epilepsy, and fragile X syndrome."

Arguably all developmental disorders that are categorized within the spectrum of specific learning difficulties have aspects of behavioural, cognitive and neurological roots. Morton and Frith (1995) suggest a causal modelling framework (CM) which draws together behavioural, cognitive and neurological dimensions, and contextualises them within the environment of the individual.

The underpinning rationale of this model is that no level should be considered independently of the other, and it should include acknowledgement of the impact of environmental influences. It is a neutral framework within which to compare theories. Frith believes that the variation in behavioural or cognitive explanations should not ignore possible common underlying factors at the biological / neurological level. In addition, epidemiological findings identify three major areas of environmental risk as socioeconomic disadvantage, socio-cultural and gender differences. Equally, complex interaction between biology and environment mean that neurological deficits will result in cognitive and behavioural difficulties, particular to the individual. CM theory has been extended by Krol et al (2004) in an attempt to explore its application to conduct disorder (Figure 2). Therefore discussion of the aetiology of dyscalculia should include a review of the literature based on a CM framework.

Whilst it could be argued that this approach sits uncomfortably close to the 'medical' rather than the 'social' model of disability, equally an understanding of biological, cognitive and behavioural aspects of dyscalculia are fundamental to the discussion of appropriate learning and teaching experiences.

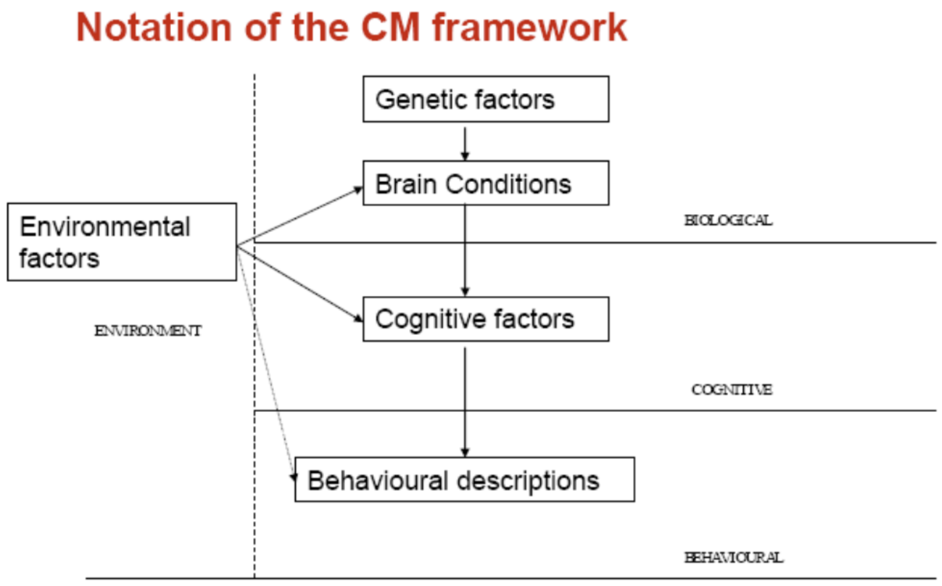


Figure 2, Causal Modelling Framework, Krol et al (2004)

Biological

Brain imaging provides clear indicators with respect to the cortical networks that are activated when individuals engage in mathematical tasks. Thioux, Seron and Pesenti (1999) state that the semantic memory systems for numerical and non-numerical information, are localised in different areas of the brain. Rourke (1993) proposes that individuals with both a mathematical and literacy disorder have deficits in the left hemisphere, whilst those exhibiting only Mathematics disorder tend to have a right hemispherical deficit;

Evidence from neuroimaging and clinical studies in brain injury support the argument that the parietal lobe, and in particular the intraparietal sulcus (IPS) in both hemispheres, plays a dominant role in processing numerical data, particularly related to a sense of the relative size and position of numbers. Cohen Kadosh et al (2007) state that the parietal lobes are essential to automatic magnitude processing, and thus there is a hemispherical locus for developmental dyscalculia. Such difficulties are replicated in studies by Ashcraft, Yamashita and Aram (1992) with children who have suffered from early brain injury to the left hemisphere or associated sub-cortical regions.

However Varma and Schwarz (2008) argue that, historically, educational neuroscience has compartmentalized investigation into cognitive activity as simply identification of brain tasks which are then mapped to specific areas of the brain, in other words '….it seeks to identify the brain area that activates most selectively for each task competency.' They argue that research should now progress from area focus to network focus, where competency in specific tasks is the product of co-ordination between multiple brain areas. For example McCrone (2002) suggests a possibility where 'the intraparietal sulcus is of a normal size but the connectivity to the "number-name" area over in Wernicke's is poorly developed.' Furthermore he states that:

> 'different brain networks are called into play for exact and approximate calculations. Actually doing a sum stirs mostly the language-handling areas while guessing a quick rough answer sees the intraparietal cortex working in conjunction with the prefrontal cortex.'

Deloche and Willmes (2000) conducted research on brain damaged patients and claim to have provided evidence that there are two syntactical components, one for spoken verbal and one for written verbal numbers, and that retrieval of simple number facts, for example number bonds and multiplication tables, depends upon format-specific routes and not unique abstract representations.

Research also indicates that Working Memory difficulties are implicated in specific Mathematics difficulties, for example Geary (1993) suggests that poor working memory resources affect execution of calculation procedures and learning arithmetical facts. Koontz and Berch (1996) found that dyscalculic children under-performed on both forward and backward digit span tasks, and whilst this difficulty is typically found in dyslexic individuals, for the dyscalculic child it tends not to affect phonological skills but is specific to number information (McLean and Hitch, 1999). Mabbott and Bisanz (2008) claim that children with identifiable Mathematics learning disabilities are distinguished by poor mastery of number facts, fluency in calculating and working memory, together with a slower ability to use 'backup procedures', concluding that overall dyscalculia may be a function of difficulties in computational skills and working memory. However it should be pointed out that this has not been replicated across all studies (Temple and Sherwood, 2002).

In terms of genetic markers, studies demonstrate a similar heritability level as with other specific learning difficulties (Kosc, 1974; Alarcon et al, 1997). In addition there appear to be abnormalities of the X chromosome apparent in some disorders such as Turner's Syndrome, where individuals functioning at the average to superior level exhibit severe dysfunction in arithmetic (Butterworth et al., 1999; Rovet, Szekely, & Hockenberry, 1994; Temple & Carney, 1993; Temple & Marriott, 1998).

Geary (2004) describes three sub types of dyscalculia: procedural, semantic memory and visuospatial, (Appendix 1). The Procedural Subtype is identified where the individual exhibits developmentally immature procedures, frequent errors in the execution of procedures, poor understanding of the concepts underlying procedural use, and difficulties sequencing multiple steps in complex procedures, for example the continued use of fingers to solve addition and subtraction problems. He argues that there is evidence that this is a left hemisphere pre-frontal brain dysfunction, that can be ameliorated or improve with age.

The Semantic memory Subtype is identified where the individual exhibits difficulties in retrieving mathematical facts together with a high error rate, For example responses to simple arithmetic problems, and accuracy with number bonds and tables. Dysfunction appears to be located in the left hemisphere posterior region, is heritable, and is resistant to remediation. The Visuospatial Subtype represents a difficulty with spatially representing numerical and other forms of mathematical information and relationships, with frequent misinterpretation or misunderstanding of such information, for example solving geometric and word problems, or using a mental number line. Brain differences appear to be located in the right hemisphere posterior region.

Geary also suggests a framework for further research and discussion of dyscalculia (Figure 1) and argues that difficulties should be considered from the perspective of deficits in cognitive mechanism, procedures and processing, and reviews these in terms of performance, neuropsychological, genetic and developmental features.

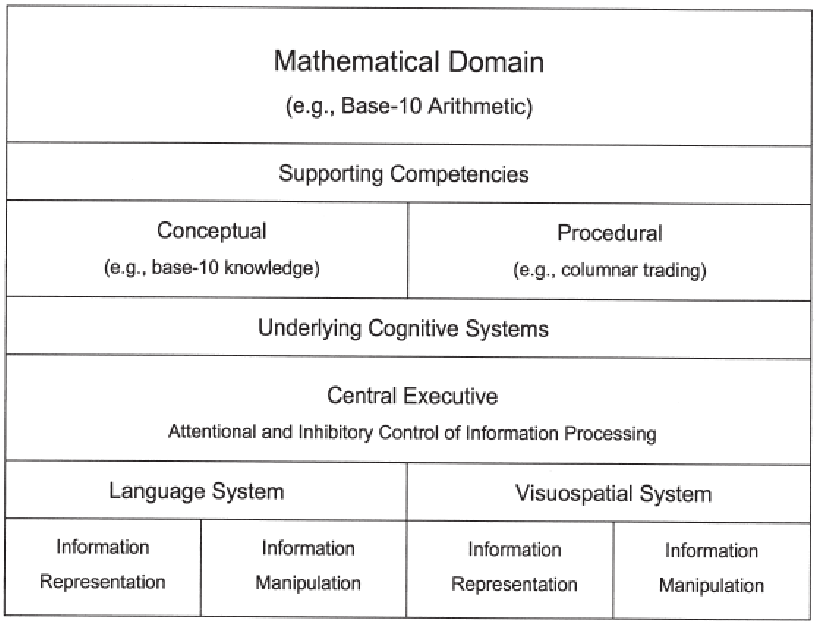


Figure 1, Geary (2004)

Investigating brain asymmetry and information processing, Hugdahl and Westerhausen (2009) claim that differences in spacing of neuronal columns and a larger left planum temporal result in enhanced processing speed. They also state that the evolution of an asymmetry favouring the left hand side of the brain is a result of the need for lateral specialisation to avoid 'shuffling' information between hemispheres, in response to an increasing demand on cognitive functions. Neuroimaging of dyslexic brains provides evidence of hemispherical brain symmetry, and thus a lack of specialisation. McCrone (2002) also argues that perhaps the development of arithmetical skills is as artificial as learning to read, which may be problematic for some individuals where the brain ‘evolved for more general purposes’.

Cognitive

Dehaene (1992) and Dehaene. and Cohen (1995, 1997) suggest a ‘triple-code’ model of numerosity, each code being assigned to specific numerical tasks. The analog magnitude code represents quantities along a number line which requires the semantic knowledge that one number is sequentially closer to, or larger or smaller than another; the auditory verbal code recognises the representation of a number word and is used in retrieving and manipulating number facts and rote learned sequences; the visual Arabic code describes representation of numbers as written figures and is used in calculation. Dehaene suggests that this is a triple processing model which is engaged in mathematical tasks.

Historically, understanding of acquisition of numerical skills was based on Piaget’s pre-operational stage in child development (2 – 7 years). Specifically Piaget argues that children understand conservation of number between the ages of 5 – 6 years, and acquire conservation of volume or mass at age 7 – 8 years. Butterworth (2005) examined evidence from neurological studies with respect to the development of arithmetical abilities in terms of numerosity – the number of objects in a set. Research evidence suggests that numerosity is innate from birth (Izard et al, 2009) and pre-school children are capable of understanding simple numerical concepts allowing them to complete addition and subtraction to 3. This has significant implications as “….the capacity to learn arithmetic – dyscalculia – can be interpreted in many cases as a deficit in the child’s concept of numerosity” (Butterworth, 2005). Butterworth provides a summary of milestones for the early development of mathematical ability based on research studies.

Age	Milestones (Typical study)
0;0	Can discriminate on the basis of small numerosities (Antell & Keating, 1983)
0;4	Can add and subtract one (Wynn, 1992)
0;11	Discriminates increasing from decreasing sequences of numerosities (Brannon, 2002)
2;0	Begins to learn sequence of counting words (Fuson, 1992); can do one-to-one correspondence in a sharing task (Potter & Levy, 1968)
2;6	Recognises that number words mean more than one ('grabber') (Wynn, 1990)
3;0	Counts out small numbers of objects (Wynn, 1990)
3;6	Can add and subtract one with objects and number words (Starkey & Gelman, 1982); Can use cardinal principle to establish numerosity of set (Gelman & Gallistel, 1978)
4;0	Can use fingers to aid adding (Fuson & Kwon, 1992)
5;0	Can add small numbers without being able to count out sum (Starkey & Gelman, 1982)
5;6	Understands commutativity of addition and counts on from larger (Carpenter & Moser, 1982); can count correctly to 40 (Fuson, 1988)
6;0	'Conserves' number (Piaget, 1952)
6;6	Understands complementarity of addition and subtraction (Bryant et al, 1999); can count correctly to 80 (Fuson, 1988)
7;0	Retrieves some arithmetical facts from memory

Figure 3, Butterworth, 2005

Geary and Hoard (2005) also outline the theoretical pattern of normal early years development in number, counting, and arithmetic compared with patterns of development seen in children with dyscalculia in the areas of counting and arithmetic.

Counting

The process of 'counting' involves an understanding of five basic principles proposed by Gelman and Gallistel (1978):

- one to one correspondence - only one word tag assigned to each counted object
- stable order - the order of word tags must not vary across counted sets
- cardinality - the value of the final word tag represents the quantity of items counted
- abstraction - objects of any kind can be counted
- order-irrelevance - items within a given set can be counted in any sequence

In conjunction with learning these basic principles in the early stages of numeracy, children additionally absorb representations of counting 'behaviour'. Children with dyscalculia have a poor conceptual understanding of some aspects of counting rules, specifically with order-irrelevance (Briars and Siegler, 1984). This may affect the counting aspect of solving arithmetic problems and competency in identifying and correcting errors.

Arithmetic

Early arithmetical skills, for example calculating the sum of 6 + 3, initially may be computed verbally or physically using fingers or objects, and uses a 'counting-on' strategy. Typically both individuals with dyscalculia and many dyslexic adults continue to use this strategy when asked to articulate 'times tables' where they have not been rote-learned and thus internalised. Teaching of number bonds or number facts aid the development of representations in long term memory, which can then be used to solve arithmetical problems as a simple construct or as a part of more complex calculation. That is to say the knowledge that 6 + 3 and 3 + 6 equal 9 is automatized.

This is a crucial element in the process of decomposition where computation of a sum is dependent upon a consolidated knowledge of number bonds. For example where 5 + 5 is equal to 10, 5 + 7 is equal to 10 plus 2 more. However this is dependent upon confidence in using these early strategies; pupils who have failed to internalise such strategies and therefore lack confidence tend to 'guess'. As ability to use decomposition and the principles of number facts or bonds becomes automatic, the ability to solve more complex problems in a shorter space of time increases. Geary (2009) describes two phases of mathematical competence: biologically primary quantitative abilities which are inherent competencies in numerosity, ordinality, counting, and simple arithmetic enriched through primary school experiences, and biologically secondary quantitative abilities which are built on the foundations of the former, but are dependent upon the experience of Mathematics instruction (Appendix 2).

In the same way that it is impossible to describe a ‘typical’ dyslexic profile, in that individuals may experience difficulties with reading, spelling, reading comprehension, phonological processing or any combination thereof, similarly a dyscalculic profile is more complex than ‘not being able to do Mathematics’. Geary and Hoard (2005) describe a broad range of research findings which support the claim that children with dyscalculia are unable to automatically retrieve this type of mathematical process. Geary (1993) suggests three possible sources of retrieval difficulties:

> ‘….a deficit in the ability to represent phonetic/semantic information in long-term memory…….. and a deficit in the ability to inhibit irrelevant associations from entering working memory during problem solving (Barrouillet et al., 1997). A third potential source of the retrieval deficit is a disruption in the development or functioning of a ……cognitive system for the representation and retrieval of arithmetical knowledge, including arithmetic facts (Butterworth, 1999; Temple & Sherwood, 2002).’

Additionally responses tend to be slower and more inaccurate, and difficulty at the most basic computational level will have a detrimental effect on higher Mathematics skills, where skill in simple operations is built on to solve more complex multi-step problem solving.

Emerson (2009) describes difficulties with number sense manifesting as severely inaccurate guesses when estimating quantity, particularly with small quantities without counting, and an inability to build on known facts. Such difficulty means that the world of numbers is sufficiently foreign that learning the ‘language of Mathematics’ in itself becomes akin to learning a foreign language.

Behavioural

Competence in numeracy is fundamental to basic life skills and the consequences of poor numeracy are pervasive, ranging from inaccessibility of further and higher education, to limited employment opportunities: few jobs are completely devoid of the need to manipulate numbers. Thus developmental dyscalculia will necessarily have a direct impact on socio-economic status, self esteem and identity.

Research by Hanich et al (2001) and Jordan et al (2003) claim that children with mathematical difficulties appear to lack an internal number line and are less skilled at estimating magnitude. This is illustrated by McCrone (2002) with reference to his daughter:

> "A moment ago I asked her to add five and ten. It was like tossing a ball to a blind man. "Umm, umm." Well, roughly what would it be? "About 50…or 60", she guesses, searching my face for clues. Add it up properly, I say. "Umm, 25?" With a sigh she eventually counts out the answer on her fingers. And this is a nine-year old.
>
> The problem is a genuine lack of feel for the relative size of numbers. When Alex hears the name of a number, it is not translated into a sense of being larger or smaller, nearer or further, in a way that would make its handling intuitive. Her visuospatial abilities seem fine in other ways, but she apparently has hardly any capacity to imagine fives and tens as various distances along a mental number line. There is no gutfelt difference between 15 and 50. Instead their shared "fiveness" is more likely to make them seem confusingly similar."

Newman (1998) states that difficulty may be described at three levels:

- Quantitative dyscalculia - a deficit in the skills of counting and calculating

- Qualitative dyscalculia - the result of difficulties in comprehension of instructions or the failure to master the skills required for an operation. When a student has not mastered the memorization of number facts, he cannot benefit from this stored "verbalizable information about numbers" that is used with prior associations to solve problems involving addition, subtraction, multiplication, division, and square roots.

- Intermediate dyscalculia – which involves the inability to operate with symbols or numbers.

Trott and Beacham (2005) describe it as:

> “a low level of numerical or mathematical competence compared to expectation. This expectation being based on unimpaired cognitive and language abilities and occurring within the normal range. The deficit will severely impede their academic progress or daily living. It may include difficulties recognising, reading, writing or conceptualising numbers, understanding numerical or mathematical concepts and their inter-relationships.
>
> It follows that dyscalculics may have difficulty with numerical operations, both in terms of understanding the process of the operation and in carrying out the procedure. Further difficulties may arise in understanding the systems that rely on this fundamental understanding, such as time, money, direction and more abstract mathematical, symbolic and graphical representations.”

Butterworth (2003) states that although such difficulties might be described at the most basic level as a condition that affects the ability to acquire arithmetical skills, other more complex abilities than counting and arithmetic are involved which include the language of Mathematics:

- understanding number words (one, two, twelve, twenty …), numerals (1, 2, 12, 20) and the relationship between them;
- carrying out mental arithmetic using the four basic arithmetical operations – addition, subtraction, multiplication and division;
- completing written multi-digit arithmetic using basic operations;
- solving ‘missing operand problems’ (6 + ? = 9);
- solving arithmetical problems in context, for example handling money and change.

Trott (2009) suggests the following mathematical difficulties which are also experienced by dyslexic students in higher education:

Arithmetical

- Problems with place value
- Poor arithmetical skills
- Problems moving from concrete to abstract

Visual

- Visual perceptual problems reversals and substitutions e.g. 3/E or +/x
- Problems copying from a sheet, board, calculator or screen
- Problems copying from line to line
- Losing the place in multi-step calculations
- Substituting names that begin with the same letter, e.g. integer/integral, diagram/diameter
- Problems following steps in a mathematical process
- Problems keeping track of what is being asked
- Problems remembering what different signs/symbols mean
- Problems remembering formulae or theorems

Memory

- Weak short term memory, forgetting names, dates, times, phone numbers etc
- Problems remembering or following spoken instructions
- Difficulty listening and taking notes simultaneously
- Poor memory for names of symbols or operations, poor retrieval of vocabulary

Reading

- Difficulties reading and understanding Mathematics books
- Slow reading speed, compared with peers
- Need to keep re-reading sentences to understand
- Problems understanding questions embodied in text

Writing

- Scruffy presentation of work, poor positioning on the page, changeable handwriting
- Neat but slow handwriting
- Incomplete or poor lecture notes
- Working entirely in pencil, or a reluctance to show work

General

- Fluctuations in concentration and ability
- Increased stress or fatigue

However a distinction needs to be drawn between dyscalculia and maths phobia or anxiety which is described by Cemen (1987) as 'a state of discomfort which occurs in response to situations involving mathematics tasks which are perceived as threatening to self-esteem.' Chinn (2008) summarizes two types of anxiety which can be as a result of either a 'mental block' or rooted in socio-cultural factors.

> 'Mental block anxiety may be triggered by a symbol or a concept that creates a barrier for the person learning maths. This could be the introduction of letters for numbers in algebra, the seemingly irrational procedure for long division or failing to memorise the seven times multiplication facts. [...] Socio-cultural maths anxiety is a consequence of the common beliefs

> about maths such as only very clever (and slightly strange) people can do maths or that there is only ever one right answer to a problem or if you cannot learn the facts you will never be any good at maths.'

According to Hadfield and McNeil (1994) there are three reasons for Mathematics anxiety: environmental (teaching methods, teacher attitudes and classroom experience), intellectual (influence of learning style and insecurity over ability) and personality (lack of self confidence and unwillingness to draw attention to any lack of understanding). Findings by Chinn (2008) indicate that anxiety was highest in Year 7 (1st year secondary) male pupils, which arguably is reflective of general anxiety associated with transition to secondary school.

Environmental

Environmental factors include stress and anxiety, which physiologically affect blood pressure to memory formation. Social aspects include alcohol consumption during pregnancy, and premature birth / low birth weight which may affect brain development. Isaacs, Edmonds, Lucas, and Gadian (2001) investigated low birth-weight adolescents with a deficit in numerical operations and identified less grey matter in the left IPS.

Assel et al (2003) examined precursors to mathematical skills, specifically the role of visual-spatial skills, executive processing but also the effect of parenting skills as an environment influence. The research measured cognitive and mathematical abilities together with observation of maternal directive interactive style. Findings supported the importance of visual-spatial skills as an important early foundation for both executive processing and mathematical ability. Children aged 2 years whose mothers directed tasks as opposed to encouraging exploratory and independent problem solving, were more likely to score lower on visual–spatial tasks and measures of executive processing. This indicates the importance of parenting environment and approach as a contributory factor in later mathematical competence.

1.3 Assessment

Shalev (2004) makes the point that delay in acquiring cognitive or attainment skills does not always mean a learning difficulty is present. As stated by Geary (1993) some cognitive features of the procedural subtype can be remediated and do not necessarily persist over time. Difficulties with Mathematics in the primary school are not uncommon; it is the pervasiveness into secondary education and beyond that most usefully identifies a dyscalculic difficulty. A discrepancy definition stipulates a significant discrepancy between intellectual functioning and arithmetical attainment or by a discrepancy of at least 2 years between chronologic age and attainment. However, measuring attainment in age equivalencies may not be meaningful in the early years of primary age range, or in the later years of secondary education.

Wilson et al (2006) suggest that assessment of developmental symptoms should examine number sense impairment. This would include:

> ‘reduced understanding of the meaning of numbers, and a low performance on tasks which depend highly on number sense, including non symbolic tasks (e.g. comparison, estimation or approximate addition of dot arrays), as well as symbolic numerical comparison and approximation’.

They add that performance in simple arithmetical calculation such as subtraction would be a more sensitive measure, as addition and multiplication is more open to compensatory strategies such as adding or counting on, and memorization of facts and sequences.

Assessment instruments

As yet there are few paper-based dyscalculia specific diagnostic. Existing definitions state that the individuals must substantially underachieve on standardised tests compared to expected levels of achievement based on underlying ability, age and educational experience. Therefore assessment of mathematical difficulty tends to rely upon performance on both standardized mathematical achievement and measurement of underlying cognitive ability. Geary and Hoard (2005) warn that scoring systems in attainment tests blur the identification of specific areas of difficulty:

> 'Standardized achievement tests sample a broad range of arithmetical and mathematical topics, whereas children with MD often have severe deficits in some of these areas and average or better competencies in others. The result of averaging across items that assess different competencies is a level of performance […] that overestimates the competencies in some areas and underestimates them in others.'

Von Aster (2001) developed a standardized arithmetic test, the Neuropsychological Test Battery for Number Processing and Calculation in Children, which was designed to examine basic skills for calculation and arithmetic and to identify dyscalculic profiles. In its initial form the test was used in a European study aimed at identifying incidence levels (see section 1.4). It was subsequently revised and published in English, French, Portuguese, Spanish, Greece, Chinese and Turkish as ZarekiR, This test is suitable for use with children aged 7 to 13.6 years and is based on the modular system of number processing proposed by Dehaene (1992).

Current practice for assessment of dyscalculia is referral to an Educational Psychologist. Trott and Beacham (2005) claim that whilst this is an effective assessment method where students present with both dyslexic and dyscalculic indicators, it is ineffective for pure dyscalculia with no co-morbidity. Whilst there is an arithmetical component in tests of cognitive ability such as the Weschler Intelligence Scale for Children (WISC) and the

Weschler Adult Intelligence Scale (WAIS), only one subtest assesses mathematical ability. Two things are needed then: an accurate and reliable screening test in the first instance, and a standardized and valid test battery for diagnosis of dyscalculia.

Standardized tests

A review of mathematical assessments was conducted through formal psychological test providers Pearson Assessment and the Psychological Corporation. The following describe tests that are either fully available or have limited availability, depending upon the qualifications of the test user.

Test of Mathematical Abilities-Second Edition (TOMA-2)
• Administration time: 60-90 minutes
• Standard scores percentiles, and grade or age equivalents providing a Mathematics quotient
• Age Range: 8 to 18.11 years
Five norm-referenced subtests, measuring performance in problems and computation in the domains of vocabulary, computation, general Information and story problems. An additional subtest provides information on attitude towards Mathematics.
Reliability coefficients are above .80 and for the Math Quotient exceed .90.

Wide Range Achievement Test 4 (WRAT 4)
• Administration time: approximately 35-45 minutes for individuals ages 8 years and older
• Standard scores percentiles, and grade or age equivalents providing a Mathematics quotient
• Age Range: 5 to 94 years
Measures ability to perform basic Mathematics computations through counting, identifying numbers, solving simple oral problems, and calculating written mathematical problems. Reliability coefficients are above .80 and for the Math Quotient exceed .90.

	Wechsler Individual Achievement Test - Second UK Edition (WIAT-II UK)
	• Administration: Individual - 45 to 90 minutes depending on the age of the examinee
	• Standard scores percentiles, and grade or age equivalents providing a Mathematics quotient
	• Age Range: 4 to 16 years 11 months. Standardised on children aged 4 years to 16 years 11 months in the UK. However, adult norms from the U.S study are available from 17 to 85 years by simply purchasing the adult scoring and normative supplement for use with your existing materials.
	Measures ability in numerical operations and mathematical reasoning. Strong inter-item consistency within subtests with average reliability coefficients ranging from .80 to .98.

	Mathematics Competency Test
	• Purpose: To assess Mathematics competency in key areas in order to inform teaching practice.
	• Range: 11 years of age to adult
	• Administration: 30 minutes – group or individual
	Key Features:
	• Australian norms • Provides a profile of mathematical skills for each student • Identifies weaknesses and strengths in Mathematics skills • Open ended question format • Helpful in planning further teaching programs • Performance based on reference group or task interpretation
	Assessment Content:
	• Using and applying Mathematics • Number and algebra • Shape and space • Handling data
	Provides a quick and convenient measure of Mathematics skills, a skills profile as well as a norm-referenced total score. The skills profile allows attainments to be expressed on a continuum from simple to complex, making the test suitable for a wide range of purposes and contexts, in schools, colleges, and pre-employment. The test utilizes 46 open-ended questions, presented in ascending order, and is easy to score. Strong reliability with internal consistency of 0.94 for the full test Validated against 2 tests with a correlation co-efficient of 0.83 and 0.80

Working memory as an assessment device

Working Memory (WM) can be described as an area that acts as a storage space for information whilst it is being processed. Information is typically 'manipulated' and processed during tasks such as reading and mental calculation. However the capacity of WM is finite and where information overflows this capacity, information may be lost. In real terms this means that some learning content delivered in the classroom is inaccessible to the pupil, and therefore content knowledge is incomplete or 'missing'. St Clair-Thompson (2010) argues that these gaps in knowledge are 'strongly associated with attainment in key areas of the curriculum'.

Alloway (2001) conducted research with 200 children aged 5 years, and claims that working memory is a more reliable indicator of academic success. Alloway used the Automated Working Memory Assessment (AWMA) and then re-tested the research group six years later. Within the battery of tests including reading, spelling and Mathematics attainment, working memory was the most reliable indicator. Similarly recent findings with children with Specific Language Impairment, Developmental Coordination Disorder (DCD), Attention-Deficit/Hyperactivity Disorder, and Asperger's Syndrome (AS) also support these claims.

Alloway states that the predictive qualities of measuring WM are that it tests the potential to learn and not what has already been learned. Alloway states that 'If a student struggles on a WM task it is not because they do not know the answer, it is because their WM 'space' is not big enough to hold all the information'. Typically, children exhibiting poor WM strategies under-perform in the classroom and are more likely to be labelled 'lazy' or 'stupid'. She also suggests that assessment of WM is a more 'culture fair' method of assessing cognitive ability, as it is resistant to environmental factors such as level of education, and socio-economic background. The current version of AWMA has an age range of 4 to 22 years.

In a review of the literature on dyscalculia, Swanson and Jerman (2006) draw attention to evidence that deficits in cognitive functioning are primarily situated in performance on verbal WM. Currently there is no pure WM assessment for adult learners, however Zera and Lucian (2001) state that processing difficulties should also form a part of a thorough assessment process. Rotzer et al (2009) argue that neurological studies of functional brain activation in individuals with dyscalculia have been limited to:

> '…….number and counting related tasks, whereas studies on more general cognitive domains that are involved in arithmetical development, such as working memory are virtually absent'.

This study examined spatial WM processes in a sample of 8 – 10 year old children, using functional MRI scans. Results identified weaker neural activation in a spatial WM task and this was confirmed by impaired WM performance on additional tests. They conclude that 'poor spatial working memory processes may inhibit the formation of spatial number representations (mental numberline) as well as the storage and retrieval of arithmetical facts'.

Computerized assessment

The Dyscalculia Screener (Butterworth, 2003) is a computer-based assessment for children aged 6 – 14 years, that claims to identify features of dyscalculia by measuring response accuracy and response times to test items. In addition it claims to distinguish between poor Mathematics attainment and a specific learning difficulty by evaluating an individual's ability and understanding in the areas of number size, simple addition and simple multiplication. The screener has four elements which are item-timed tests:

1. Simple Reaction Time

Tests of Capacity:
2. Dot Enumeration
3. Number Comparison (also referred to as Numerical Stroop)

Test of Achievement:
4. Arithmetic Achievement test (addition and multiplication)

Speed of response is included to measure whether the individual is responding slowly to questions, or is generally a slow responder.

The Mathematics Education Centre at Loughborough University began developing a screening tool known as DyscalculiUM in 2005 and this is close to publication. The most recent review of development was provided in 2006 and is available from http://Mathematicstore.gla.ac.uk/headocs/6212dyscalculium.pdf The screener is now in its fourth phase with researchers identifying features as:

- Can effectively discriminate dyscalculia from other SpLDs such as Asperger's Syndrome and ADHD
- Is easily manageable
- Is effective in both HE and FE
- Can be accommodated easily into various screening processes
- Has a good correlation with other published data, although this data is competency based and not for screening purposes
- Can be used to screen large groups of students as well as used on an individual basis

1.4 Incidence

The lack of consensus with respect to assessment and diagnosis of dyscalculia, applies equally to incidence. As with dyslexia, worldwide studies describe an incidence ranging from 3% to 11%, however as there is no formalised method of assessment such figures may be open to interpretation.

Research by Desoete et al (2004) investigated the prevalence of dyscalculia in children based on three criterion: discrepancy (significantly lower arithmetic scores than expected based on general ability), performance at least 2 SD below the norm, and difficulties resistant to intervention. Results indicated that of 1, 336 pupils in 3rd grade (3rd class) incidence was 7.2% (boys) and 8.3% (girls), and of 1, 319 4th grade (4th class) pupils, 6.9% of boys and 6.2% of girls.

Koumoula et al. (2004) tested a sample population of 240 children in Greece using the Neuropsychological Test Battery for Number Processing and Calculation in Children, and a score of <1.5 SD was identified in 6.3% of the sample. Findings by Von Aster and Shalev (2007) in a sample population of 337 Swiss children reported an incidence of 6.0 % using the same assessment method and criterion.   Mazzocco and Myers (2003) used multiple tests of arithmetic skills (Key Math Subtests, Test of Early Math Ability, and Woodcock-Johnson Revised Math Calculations) together with a criterion of persistent diagnosis across more than one school year. Incidence rates for 3rd grade children fell between 5% and to 21%.

Findings from cross-cultural studies indicate that incidence is more prevalent in boys than girls, the risk ratio being 1.6 to 2.2. In terms of co-morbidity with other specific learning difficulties, studies by Gross-Tsur et al (1996), Barbaresi et al (2005) and Von Aster and Shalev (2007) provide evidence of a coexisting reading difficulty, the percentages across all three studies falling at 17%, 56.7% and 64%.   Additionally, a greater number of children with dyscalculia exhibit clinical behaviour disorders than expected.

Barbaresi et al (2005) investigated the incidence of Mathematics learning disorder among school-aged children, via a population-based, retrospective, birth cohort study. The research study used a population sample of all children born between 1976 and 1982. Data was extracted from individually administered cognitive and achievement tests together with medical, educational, and socioeconomic information. Findings identified a cumulative incidence rate of Mathematics disorder by age 19 years within a range of 5.9% to 13.8%. The results suggest that dyscalculia is common among school children, and is significantly more frequent among boys than girls. This level of incidence reflects a similar incidence of dyslexia, which is identified as being between 4% and 10% of the population.

1.5 Intervention

At a neurological level, St Clair-Thompson (2010) states that remediation of WM would enhance performance in academic progress. She suggests that memory strategy training and practice in memory tasks are effective intervention tools. This might include adjustments to the teaching environment such as repetition of material in a variety of formats, breaking down tasks into smaller units, and use of memory techniques. Research into the use of computer programmes such as ‘Memory Booster’ (Leedale et al, 2004) whilst demonstrating improved WM performance, does not confirm that they can enhance or improve academic attainment (St Clair-Thompson et al, 2010; Holmes et al, 2009).

Wilson et al (2006) developed and trialled software designed to remediate dyscalculia, called ‘The Number Race’. The underlying rationale of this system is the presence of a "core deficit" in both number sense and accessing such a sense through visual symbolic representation. The programme claims to remediate difficulties using mathematical problems which are adaptive to the age and ability level of the child. The software was piloted with a small sample of 7–9 year old French children with mathematical difficulties, for 30 minutes a day over 5 weeks. Children were tested pre and post intervention on tasks measuring counting, transcoding, base-10 comprehension, enumeration, addition, subtraction, and symbolic and non-symbolic numerical comparison. Whilst the sample exhibited increased performance on core number sense tasks such as subtraction accuracy, there was no improvement in addition and base-10 comprehension skills. However this is the first step in a series of clinical trials to build on this programme.

Sharma (1989) argues that Mathematics should be considered as a separate, symbolic ‘language’ system and teaching should reflect this. Specifically, that terminology, vocabulary and syntax of mathematical language must be taught strategically to ensure understanding of mathematical concepts, to underpin learning of mathematical methods. Sharma also makes the point that consideration should be given to inclusive teaching principles, methods and materials to address difficulties at every level. She suggests five critical factors in delivering the Mathematics curriculum effectively:

1. Assessment of mathematical knowledge and strategies used by the learner to determine teaching methodology.

2. Assessment and identification of learning style (whether quantitative or qualitative) and recognition that this is unique to the individual. For example quantitative learners may favour learning the procedural aspect of Mathematics, and to deduce answers from having learned general mathematical principles. Qualitative learners are more dependent upon seeing parallels and relationships between elements.

3. Assessment of seven ‘pre-Mathematics’ skills:

- Sequencing
- Direction and laterality
- Pattern recognition
- Visualisation
- Estimation
- Deductive reasoning
- Inductive reasoning

4. Specific teaching of mathematical language and syntactical variations, for example that 33 – 4 is the same as ‘subtract 4 from 33’ and 4 less than 33’.

5. A systematic approach to the introduction and teaching of new mathematical concepts and models.

A detailed discussion of these factors is available in Appendix 4. The consensus on guidelines for effective intervention can be summarized as follows:

1. Enable visualization of Mathematics problems. Provide pictures, graphs, charts and encourage drawing the problem.

2. Read questions / problems aloud to check comprehension. Discuss how many parts / steps there may be to finding the solution.

3. Provide real life examples.

4. Ensure that squared / graph paper is used to keep number work and calculation.

5. Avoid fussy and over-detailed worksheets, leave space between each question so that pupils are not confused by questions that seem to merge together.

6. Teach over-learning of facts and tables, using all senses and in particular rhythm and music. Warning: meaningless repetition to learn facts off by heart does not increase understanding.

7. Provide one-to-one instruction on difficult tasks. If a pupil does not understand, re-frame and re-word the question / explanation

8. Use a sans serif font in minimum 12 point.

9. Provide immediate feedback and provide opportunities for the pupil to work through the question again. Encourage opportunities to see where an error has occurred.

10. In early stages of Mathematics teaching, check that the pupil has understood the syntactical variations in Mathematics language.   Encourage the pupil to verbalize the problem stages, for example: 'To do this I have to first work out how many thingies there are and then I can divide that number by the number of whatsits to find out how many each one can have.'

11. Allow more time to complete Mathematics work.

12. Ask the pupil to re-teach the problem / function to you.

Whilst Sharma (1989) highlights the language of Mathematics as key in the building of foundation skills, critically, in the NCCA Report (2005) only 17.2% or primary teachers identified the use of Mathematics language as an effective strategy in the teaching of Mathematics skills, and only 10.7%

reported linking Mathematics activities to real life situations. Butterworth (2009) suggests four basic principles of intervention:

- Strengthen simple number concepts

- Start with manipulables and number words

- Only when learner reliably understands relationship between number words and concrete exemplars, progress to numeral symbols

- Structured teaching programme designed for each learner

Technological aids tend to be limited to tool such as calculators, which include talking calculators and enlarged display screens, buttons and keypads. There are a plethora of computer programmes (Appendix 5) on the market which claim to improve the underlying cognitive skills associated with reading, spelling and number. However caution should be exercised with regard to computerized training. Owen et al (2010) researched the efficacy of brain training exercises conducting an online study with more than 11,000 participants. Whilst performance of all participants in improved over time on the experiment, re-testing on the initial performance tests indicated that 'these benefits had not generalised, not even when the training tests and benchmark tests involved similar cognitive processes'.

Section 2: Accessing the curriculum

2.1 Primary schools programme

The National Council for Curriculum and Assessment (NCCA) 'Primary Curriculum Review' (2005) reported that 66.4% of teachers hardly ever or never used diagnostic tests, and 77.2% hardly ever or never used standardised tests as a means for assessing performance in Mathematics (Figure 4). Whilst it is acknowledged that such tests do not play a role in supporting the teaching and learning process, arguably they are necessary to monitor the progress – or lack of – for pupils who are exhibiting difficulty in accessing the Mathematics curriculum.

Table 4.13 Use of assessment tools in the Mathematics Curriculum

	hardly ever/never		once or twice a month		at least a few times a week		Total	
	n	%	n	%	n	%	n	%
Teacher observation	2	**0.3**	4	**.6**	646	**99.1**	652	**100.0**
Teacher-designed tasks and tests	9	**1.4**	159	**24.4**	484	**74.2**	652	**100.0**
Work samples, portfolios and projects	112	**17.8**	202	**32.2**	314	**50.0**	628	**100.0**
Curriculum profiles	336	**61.3**	162	**29.6**	50	**9.1**	548	**100.0**
Diagnostic tests	359	**66.4**	145	**26.8**	37	**6.8**	541	**100.0**
Standardised tests	427	**77.2**	105	**19.0**	21	**3.8**	553	**100.0**

n=719

Figure 4, NCCA, 2005

The report identified 20.9% of teachers as stating that standardized tests were unsuitable 'to assess specific learning disability child in comparison to mainstream'. In addition they were of the opinion that there was an 'over-reliance on written assessment' Of the 459 teachers who responded to the challenges of assessing Mathematics ability, over 80% of this number stated that primary difficulties in assessment were time, the range of Mathematics abilities amongst pupils, appropriate assessment tools and language.

With respect to time, teachers stated that large class sizes were a contributing factor to difficulties particularly ‘time constraints for assessing children with learning difficulties’. For classes with a wide range of ability level, difficulties were expressed in assessing ‘how precisely each child coped with a new concept’, ‘pinpointing [their] specific mathematical difficulty’, and ‘tailoring test to individuals to pinpoint areas of weakness’. With respect to standardised assessment tools, 20.9% of teachers felt they were inadequate for testing performance against the revised curriculum and that they were inappropriate to ‘assess specific learning disability in comparison to the mainstream.’

A critical point was made relating mathematical ability and language ability, supporting Sharma’s (1989) assertions, with 7.1% of teachers observing ‘mathematical language itself to be problematic for certain children’ and that ‘lack of expressive language for Mathematics’ is a factor in difficulties. Clearly then, there are practical constraints in assessing Mathematics performance, which is a cause for concern. If a specific difficulty in Mathematics is not identified during the early years of education where a solid mathematical foundation is constructed, such difficulties will multiply exponentially.

2.2 Secondary programme

In September 2003 the NCCA introduced plans for the Project Mathematics programme. This is a school-based initiative which aims to address issues such as school completion targets, and access to and participation in third level through changes to the Junior (JC) and Senior cycle (LC) Mathematics curriculum. Objectives include a greater focus on the learner’s understanding of key Mathematics skills, the role of Mathematics assessment, and the contribution of such skills to Ireland as a knowledge economy. To achieve these aims and objectives the project is committed to getting teachers involved in changes to the curriculum, by encouraging lesson development, adaptation and refinement that will feedback into the curriculum development process. In terms of curriculum structure there will be incremental revisions to syllabi, and an assessment approach which reinforces these changes.

In 2006 the NCCA conducted a review of Mathematics in Post-Primary Education which included the following remarks:

> ‘The difficulties that students experience in Third Level are due to mathematical under-preparedness in terms of mathematical knowledge and skills as well as attitudes. The Leaving Certificate Ordinary Level course is not working well for students in this regard and needs attention.
>
> The examination needs to be less predictable. At the moment it seems easy for teachers / students / media to predict the format of the paper and even the individual questions. The fact that ‘question 1 is always about topic X’ reinforces the notion that rote learning is the way to score highly’.

If this is the case then we could assume that some pupils with dyscalculia may be more successful in that they can revise to a set pattern, and anecdotally, this appears to be a strategy that is widely used.

Project Mathematics aims to introduce a number of new initiatives: In the first instance a bridging framework between primary and secondary level is proposed. This will take the form of a common introductory course in first year of secondary, with the purpose of building on the knowledge, understanding and skills developed at primary school. For this reason choice of syllabus level at Foundation, Ordinary or Higher level will be delayed choice. With respect to the JC years there will be two syllabus levels, ordinary and Higher Level, with a Foundation Level examination based on the Ordinary syllabus. The uptake targets are that at least 60% of the JC cohort will study at Higher Level.

Planned syllabus changes at junior cycle and senior cycle includes 5 strands:

- Statistics and probability
- Geometry and trigonometry
- Number
- Algebra
- Functions

The project began with the introduction of strands 1 and 2 into 24 schools in September 2008. These schools continued to add two new strands (3 & 4) in September 2009. The programme for all other schools will commence from September 2010, and a programme of professional development for Mathematics teachers will begin this autumn.

In April 2010, Mary Coughlan, Minister for Education, announced that a scheme of bonus points would be introduced to encourage pupils to pursue Higher Level Mathematics in the Leaving Certificate. John Power, Director General of Engineers Ireland welcomed this suggestion, whilst acknowledging that in isolation it would not provide a solution, but that specific training and qualification in Mathematics for teachers is fundamental. The Royal Irish Academy (2008) state that only 20**%** of teachers of Mathematics studied the subject beyond the first year of their primary degree, and DES (2006) findings indicate that 70**%** of school inspectors describe teachers' knowledge of methods of teaching Mathematics as 'somewhat limited.' Research by Ni Riordain and Hannigan (2009) found that 48**%** of Mathematics teachers in post-primary schools 'have no qualification in Mathematics teaching.'

2.3 Intervention

Travers (2010) discusses inequitable access to the Mathematics curriculum and the implications for provision of learning support within Irish primary schools. Travers argues that the general allocation model of learning support, and subtle changes to the wording of DES guidelines on the provision of learning support, are constraining access to early intervention. He points out that intervention is targeted at 'pupils who are performing at or below the 10th percentile on nationally standardized tests' with wording amended from 'English and / or Mathematics' to 'English or Mathematics', implying that intervention is available for one but not both area of difficulty (DES, 2000). Travers further points out that the 2005 inspection of literacy and numeracy provision / achievement in disadvantaged schools, there was a significant shortfall in provision of learning support in numeracy. Surgenor and Sheil (2010) examined differences in learning support provision for English and Mathematics across 172 Irish primary schools. Only 3% of schools provided intervention support purely in Mathematics, compared to over 33% of schools providing support in English.

Literature on quantity and quality of support for Mathematics in Irish schools indicates that substantial increases in learning support staff, concrete resources, quality of teacher training, curriculum structure and timetabling are urgently required. The shortfall in provision is illustrated by the rising demand for 'grinds' services in Mathematics at primary level, accessible only to those parents with the requisite financial resources. It is clear that plugging holes in the secondary curriculum is ineffective in the long term, and that a 'bottom up' rather than 'top down' approach is required, in that intervention schemes must address Mathematics from the early years of education. In October 2009, the NCCA published *Aistear: the Early Childhood Curriculum Framework* aimed at primary age children from birth to 6 years. This scheme is targeted at parents, teachers and other professional practitioners with an emphasis on communication and learning through language in every subject area.

Engineers Ireland (2010) propose a 10 point action plan to address the question of Mathematics and future performance at primary, secondary and tertiary level (Appendix 6). They also stress the advantages of a bottom up approach, and provide suggestions for greater accessibility and flexibility in providing Mathematics support, specifically:

- The need to foster interest in Mathematics at both primary and secondary level, and in particular within the Transition Year programme.

- Harnessing the power of ICT to contextualise the teaching of Mathematics and Science at Primary and Second Level.

- Construction of a Wiki-Solution web page to assist students with problem solving in Mathematics and Applied Mathematics.

Section 3: Transition to third level

3.1 Performance in State Examinations

In order to matriculate to an Irish university, students must meet specific minimum entry requirements for each institution of higher education. Currently these are:

National University of Ireland: 6 subjects, including English, Irish and a third language. Students must have achieved grade C at Higher Level. in two of these subjects.

University of Limerick: 6 subjects, including English, Irish and a third language. Students must have achieved grade C at Higher Level in two of these subjects.

Trinity College Dublin: 6 subjects, with grade C on 3 Higher Level papers and a pass in English, Mathematics and another language.

Dublin City University: 6 subjects, with a grade C on 2 Higher-Level papers and a pass in Mathematics and either English or Irish.

Institutes of technology
Honours Degree courses: grade C in 2 subjects at Higher Level and grade D in 4 other subjects, including Mathematics and Irish/English.
Higher Certificate and Ordinary Degree courses: 5 grade Ds, including Mathematics and Irish/English
.
Colleges of education: 3 grade Cs on Higher-Level papers, including Irish, and three grade Ds, including Mathematics and English.

A pass means grade D or above on Ordinary or Higher papers. A significant number of pupils do not matriculate with a Leaving Certificate as a result of failing the Mathematics examination. Oldham (2006) states that:

> 'The percentage of students obtaining low scores (grade E, grade F, or no grade) in Mathematics in the Ordinary-level Leaving Certificate examination in particular means that some thousands of students leave the school system each year without having achieved a grade regarded as a `pass' in Mathematics. Such students are in general excluded from third-level courses that require mathematical knowledge and skills.'

Mac an Bhaird (2008) discusses factors associated with poor Mathematics performance:

> 'However, some of the main factors listed in [Lyons et al, 2003] and elsewhere include bad publicity for Mathematics, negative attitudes towards the subject, the high percentage of second-level students who go onto third-level, the socio-economic background of the student, increased competition for places, pressure on students and teachers to achieve the highest possible points, little understanding of the context or background of Mathematics, little appreciation of the applications of Mathematics in everyday life, rote learning by heart, etc.'

The State Examinations Commission provides annual and cumulative statistics indicating performance in Leaving Certificate Mathematics on Foundation, Ordinary and Higher level papers. Results are available in two formats: as a percentage breakdown of candidates by grade awarded in each subject, and percentage / number breakdown of results by gender across all levels. Annual Leaving Certificate statistics were downloaded from www.examinations.ie for the period 2001 - 2009.

These results were collated and re-tabulated, and a comparative analysis was conducted across all grades for Foundation, Ordinary and Higher results between 2001 and 2008 (statistics for 2009 are currently provisional and thus were not included). A full analysis of these statistics is available in Appendix 1. Findings indicated that the number of pupils who failed to matriculate in Mathematics between 2001 and 2008, and who were therefore prevented from transitioning to college is 43, 892. In 2008 alone, 5,049 students failed to matriculate. It is worth noting that whilst there are fluctuations in performance for A – D grades at all levels for the period 2001 – 2008, the number of students failing to matriculate having achieved E, F and No Grade is fairly consistent (Figure 5).

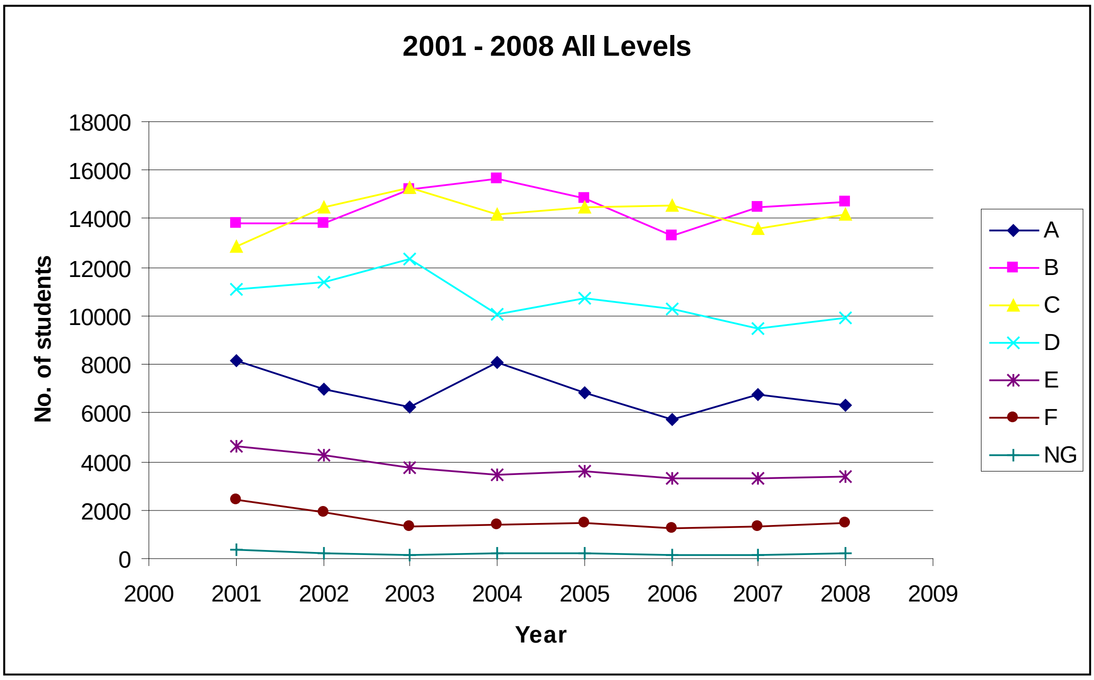
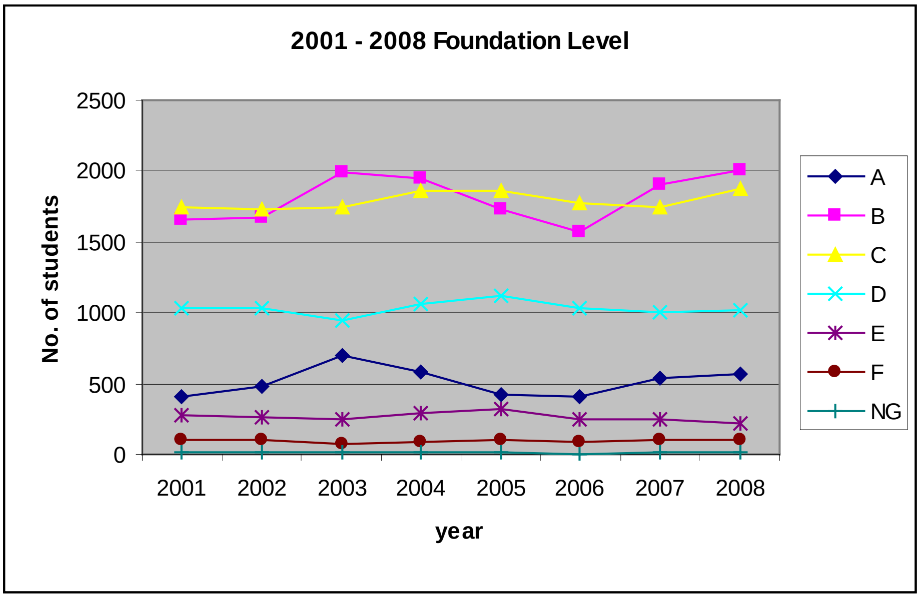
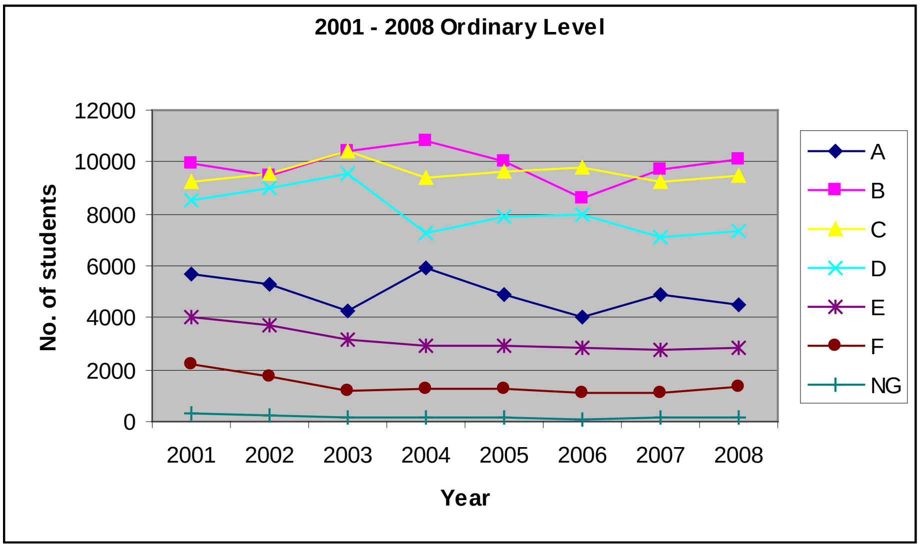


Figure 5, grade comparison 2001 - 2009

This can be further illustrated by examining differences in performance for each of the three levels of examination paper (Foundation, Ordinary and Higher) separately, for the same period (Figure 6). Clearly, whilst there are achievement fluctuations across the years for grades A to D, there is a curiously 'flat' effect across E, F and No Grade results.





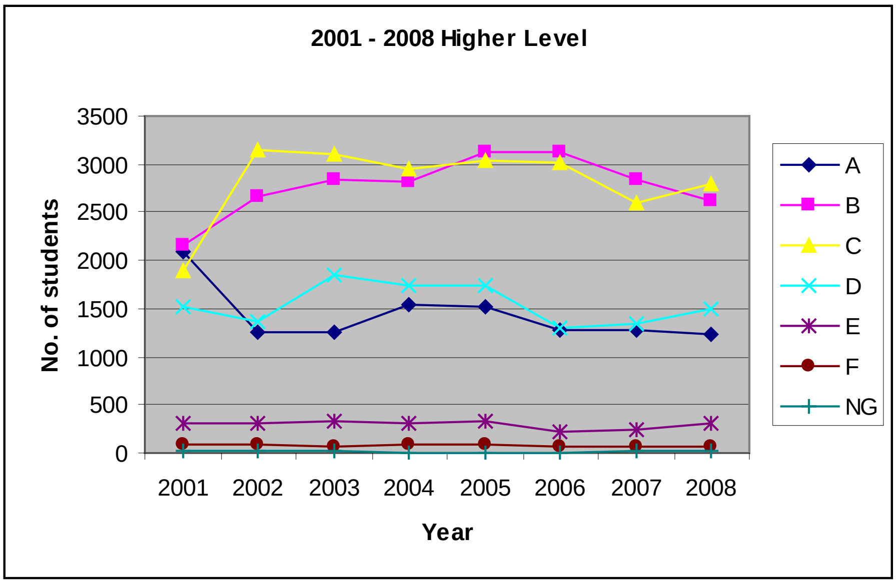
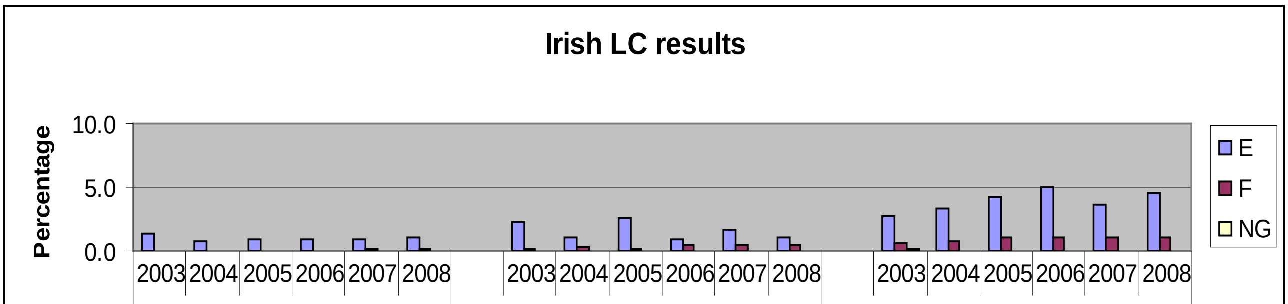


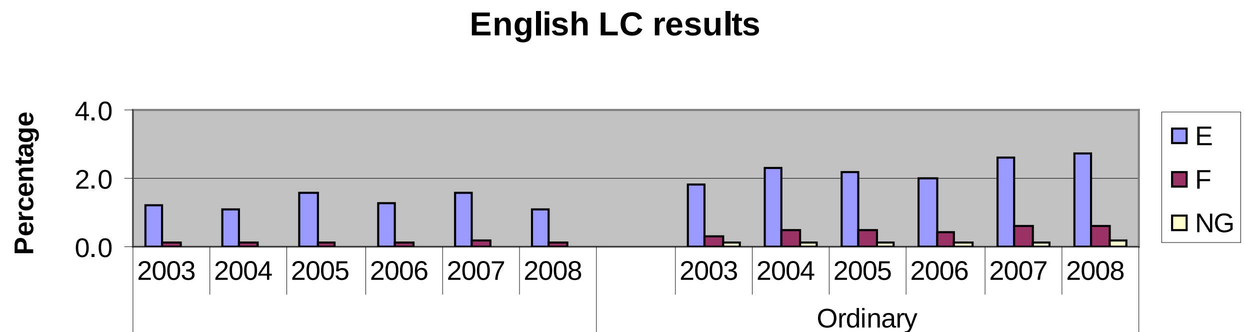
Figure 6, grade comparison for each level 2001 - 2008

There are a number of questions that need to be asked in view of these figures, not the least of which is, what do they mean? Why does the percentage of pupils achieving E, F and NG remain reasonably constant across the timeframe of 9 years, compared with pupils achieving higher grades, and why is this the case across all three papers? Do they represent pupils with mild general learning difficulties? Are they representative of pupils who have not received adequate teaching and supports in Mathematics? Or are they in fact pupils with an undiagnosed specific learning difficulty in Mathematics? One further question to be investigated is how this compares with the two remaining core curriculum subjects, Irish and English.

Comparative statistics were extracted from SEC results for the five year period between 2003 and 2008 in the three core curriculum subjects and across all levels of papers. It is clear from the tables below that there is a discrepancy between the percentages for the lowest grades achieved in both Irish and English, compared with those in Mathematics, particularly at Foundation level (Figure 8).

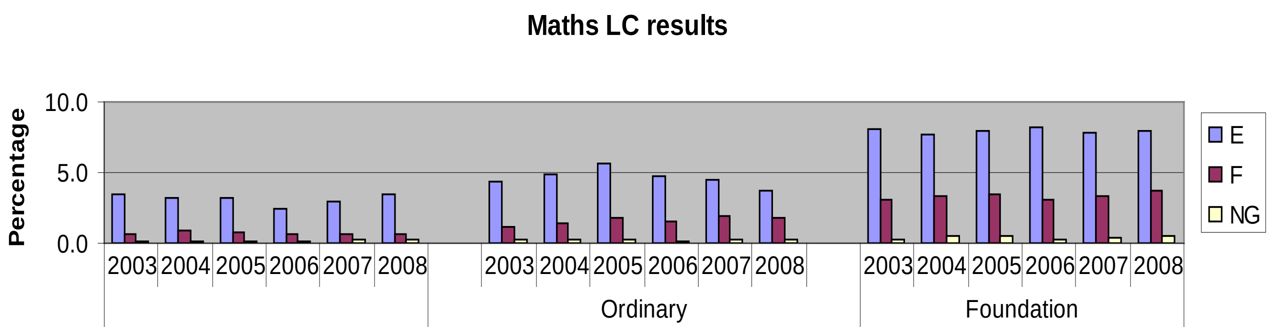


	2003	2004	2005	2006	2007	2008	2003	2004	2005	2006	2007	2008	2003	2004	2005	2006	2007	2008
							Ordinary						Foundation					
E	1.3	0.7	0.9	0.9	0.9	1.0	2.2	1.0	2.6	0.9	1.7	1.0	2.8	3.3	4.3	5.0	3.7	4.5
F	0.0	0.0	0.0	0.0	0.1	0.1	0.2	0.3	0.2	0.4	0.5	0.4	0.6	0.7	1.0	1.1	1.0	1.0
NG	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0



	2003	2004	2005	2006	2007	2008		2003	2004	2005	2006	2007	2008
								Ordinary					
E	1.2	1.1	1.6	1.3	1.6	1.1		1.8	2.3	2.2	2.0	2.6	2.7
F	0.1	0.1	0.1	0.1	0.2	0.1		0.3	0.5	0.5	0.4	0.6	0.6
NG	0.0	0.0	0.0	0.0	0.0	0.0		0.1	0.1	0.1	0.1	0.1	0.2

Year



	2003	2004	2005	2006	2007	2008		2003	2004	2005	2006	2007	2008		2003	2004	2005	2006	2007	2008
								Ordinary							Foundation					
E	3.5	3.2	3.2	2.5	2.9	3.5		4.3	4.9	5.7	4.8	4.5	3.7		8.1	7.7	7.9	8.2	7.8	8.0
F	0.6	0.9	0.8	0.7	0.7	0.7		1.2	1.4	1.8	1.6	1.9	1.8		3.1	3.3	3.5	3.1	3.3	3.7
NG	0.1	0.1	0.1	0.1	0.2	0.2		0.2	0.2	0.2	0.1	0.3	0.2		0.3	0.5	0.5	0.3	0.4	0.5

Year

Figure 8, core subject comparison, all levels 2003 - 2008

The percentage ranges for all subjects can be summarized as below:

Subject (all levels		Range of percentages	
	No Grade	F	E
Irish	0.0 – 0.1	0.0 – 1.1	0.7 – 5.0
English	0.0 - 0.2	0.1 - 0.6	1.1 - 2.7
Mathematics	0.1 – 0.5	0.6 – 3.7	2.5 – 8.0

It could be suggested that this may be reflective of two factors: pupils who have been granted an exemption from spelling and written punctuation elements in English who might otherwise have failed to achieve a pass; pupils who have been awarded an exemption from examination in Irish, who might otherwise have failed to achieve a pass in that subject. As there is no equivalent accommodation for Mathematics this may well be a reason for this discrepancy, and thus an argument for equivalent recognition of the need for reasonable accommodations in state examinations for specific learning difficulty in Mathematics.

If the incidence of dyscalculia is taken as between 4% and 10% of a sample population, then potentially the number of pupils with dyscalculia in this same period might be estimated to be between 202 and 504.

Year	Total number of students who failed to matriculate	Potential number of students at risk with dyscalculia	
		4% incidence	10% incidence
2001	7402	**296**	**740**
2002	6409	**256**	**640**
2003	5211	**208**	**521**
2004	5096	**204**	**509**
2005	5270	**210**	**527**
2006	4697	**188**	**469**
2007	4758	**190**	**475**
2008	5049	**202**	**504**

Of equal importance is what happens to these pupils who fail to matriculate? What do they do and where do they go, and has this information been compiled? Some pupils evidently repeat the Leaving Certificate to gain a higher grade in Mathematics, some pupils feed into courses which do not require matriculation, and it is likely that a number apply as mature students. To investigate this latter circumstance, a sweep was conducted of mature students registered with the Disability Service over the last 3 years.

A total of 144 mature students were registered on undergraduate courses, with an age range of 24 to 68 years. Academic records were checked and those who matriculated the Leaving Certificate and those for whom no archive record was found, were removed. Of the remaining 95 students, 19 failed Mathematics and thus did not matriculate. The remaining 76 students had no Leaving Certificate result recorded, possibly because they left school after completing Junior Cycle education. The 19 students failing to matriculate in Mathematics are all registered with the Disability Service as having a specific learning difficulty, and their performance in Mathematics LC is as follows:

Higher level	Ordinary level	Grade E	Grade F	NG
1	18	12	4	3

This represents 20% of mature students registered with the service for whom academic records were available. Investigation of academic performance in senior cycle education for all mature students would illuminate whether these findings can be generalised.

3.2 Access through DARE process

Applicants to DARE are invited to submit a personal statement recoding their particular difficulties. The incidence of dyscalculia is significantly under-represented in this cohort.

Year	Total no. of applicants with SpLD	Total no. declaring dycalculia	Total no. providing evidence (documents)
2008	1,223	3	1
2009	1,929	9	2
2010	1,398	3	2

In 2008 only 3 applications to DARE were formally submitted on the grounds of dyscalculia, and of those only 1 provided documentation confirming dyscalculia. In 2009 only 9 applications were formally submitted on the grounds of dyscalculia, including one application which also stated a hearing impairment, and of those only 2 provided documentation that included any attainment scores; none specifically stated dyscalculia. In this same year only three applicants in 2009 described dyscalculia as affecting performance:

> "[It] has affected my academic potential primarily in Mathematics, where I have significant difficulties in a number of areas. I often struggle to keep up with the class and maintain the levels required. Consequently I spend much longer than the average student studying Mathematics. I also have great difficulty memorizing things such as ……. times tables. "

> "The learning disability dyscalculia has had a huge impact on my academic potential. Apart from Mathematics where the effect is severe it also impacts on music theory and any subject that involves counting. My concept of time is greatly inhibited also."

> "From a young age it was discovered that I had hearing problems. I therefore missed hearing vital sounds needed for language and speech development…... My hearing problem was a huge factor in discovering

> I had dyslexia. I had missed the basics in Language and Mathematics as I either did not understand them or just did not hear it. This made my school life difficult as the higher up in school I went the more challenging it was which meant I missed out on more studies. In 4th grade my work was below grade level and I had problems with Mathematics."

In 2010 only 3 applicants formally applied for consideration under dycalculia, and only one applicant specifically described their difficulties in Mathematics:

> 'I have struggled with Mathematics throughout school. I have difficulty knowing what to do with Mathematics problems but I find that when I know implicitly I am fine. I can follow Mathematics methods step by step but I need to practice them over and over again. I find it extremely difficult to calculate in my head. I have difficulty learning multiplication tables, common sequences, telephone numbers and number sequences. I reverse numbers and symbols.'

CAO applicants with a disability have the option of disclosing a disability prior to entering college. Of the 546 who disclosed and applied to Trinity College in 2008, 10 applicants failed to matriculate in Mathematics, only 1 of whom had reported scores that identified them as having a difficulty in Mathematics, reading and spelling. Of the 659 who disclosed and applied to Trinity College in 2009, 10 applicants failed to matriculate in Mathematics, only 3 having reported attainment scores that identified them as having a difficulty in Mathematics, reading and spelling. There appears to be a lack of both recognition and adequate assessment for a specific difficulty with Mathematics.

3.3 Implications for transition to higher education

The Faculty of Engineering, Mathematics and Science in Trinity College Dublin hosted a Mathematics symposium in March 2010 entitled 'The Place of Mathematics Education in Ireland's Future'. The purpose of the symposium was to review issues with regard to Mathematics curriculum at second level, and review the proposal by some universities to re-introduce bonus points for Higher Level (HL) Mathematics at Leaving Certificate. Presentations delivered at this symposium are available at http://www.tcd.ie/ems/Mathematics-symposium/presentations

Elizabeth Oldham (TCD) stated that less time than previously is given to Mathematics in 1st and 2nd year, so time pressures in covering the syllabus, limited resources and insufficient opportunities to do 'up close' work with students results in pre-selection of Mathematics topics. She also notes that some teachers of other subjects are asked to teach Mathematics. This is problematic in that they may have limited specialised knowledge of the subject, with the result that rules are taught 'without reasons'. She adds that Foundation level is not recognised as providing a bedrock of mathematical understanding and competence, and thus students opt for Ordinary level, reinforcing a culture of rote learning and teaching. It is this method of 'shortcut' Mathematics that students bring with them to third level.

Maria Meehan (UCD) further emphasised the necessity for strong mathematical foundations, stating that 'Emphasis must be placed on the understanding of mathematical concepts. "Teaching for understanding" and "learning with understanding" takes time. Valuing understanding can result in students' development of mathematical skills' Meehan also makes the point that students / teachers need to recognize that Mathematics underpins many Arts disciplines, and is not only necessary for achievement in academic courses, but are an integral part of life skills.

Trinity Mathematics Waiver

Current admissions policy in Trinity stipulates that on the basis of information contained in the evidence of a specific learning difficulty, the Disability Service may recommend to that a matriculation requirement may be waived (the modern language in the case of an applicant who has dyslexia or a hearing impairment). Additionally, policy also states that in no circumstances will a specific course requirement (for example Higher Level Leaving Certificate Mathematics grade C3 for Engineering, or a language requirement specified for a particular course) be waived.

However, having conducted an extensive review of Mathematics difficulties, the Disability Service proposed that as part of the College Matriculation requirements that Mathematics be open to a waiver under very specific circumstances, such as students with dyscalculia or students who are blind and who can demonstrate that they had limited access to the Mathematics curriculum. It is not the intention that a Mathematics waiver be granted where there is a mathematical requirement as a core component of a degree course, for example within programmes such as Business, Sociology and Psychology.

In September 2009 Trinity introduced a Mathematics Waiver whereby students may apply for a waiver of the Mathematics requirement if they function intellectually at average or above average level, and have a specific learning difficulty (dyscalculia) of such a degree of severity that they fail to achieve expected levels of attainment in basic skills in Mathematics. Such evidence must be provided by a fully qualified psychologist. It is hoped that this initiative will enable students who might otherwise have been prevented from participating in higher education.

3.4 Mathematics support in higher education

In 2009 Qualifax initiated an enquiry into the range of Mathematics support provided to students in third level institutions. Colleges were asked to provide details on access to specialized teaching and Mathematics assessment. Responses which are summarized in the table below described initiatives that include Mathematics programmes available to second level students, 'second chance' admissions routes for students who did not matriculate in Mathematics, and ongoing Mathematics support within college via a range of strategies. Full text of these responses is available from http://www.qualifax.ie/index.php?option=com_content&view=article&id=122&Itemid=183.

Of the 17 institutions surveyed all provided some form of Mathematics support or advice, whether that be in the form of a dedicated centre, individual tutorials or peer support. Only the NCI, IT Tallaght and IT Tralee provided outreach to second level schools; the latter also engages in a pre-college Mathematics skills course, Headstart. Many colleges permit the sitting of a special Mathematics examination for prospective Engineering students, and there are two interesting and unique approaches adopted by the American University and Letterkenny IT. The American University will permit Business students who did not matriculate in Mathematics to complete the first year of the undergraduate course whilst preparing to re-take Math LC in the following year. Letterkenny IT provide an Intensive Mathematics programme and subsequent examination which permit students to apply for any vacant places.

Mac an Bhaird (2008) investigated the rationale and necessity for Mathematics support in higher education. His paper reviews the information collated by the Regional Centre for Excellence in Mathematics Teaching and Learning (CEMTL) in the University of Limerick from all Mathematics support facilities in Ireland. In addition he briefly discusses factors associated with poor Mathematics performance at third level:

> 'However, some of the main factors listed in [Lyons et al, 2003] and elsewhere include bad publicity for Mathematics, negative attitudes towards the subject, the high percentage of second-level students who go onto third-level, the socio-economic background of the student, increased competition for places, pressure on students and teachers to achieve the highest possible points, little understanding of the context or background of Mathematics, little appreciation of the applications of Mathematics in everyday life, rote learning by heart, etc.'

The Eureka Centre is hosted by the University of Loughborough http://eureka.lboro.ac.uk.html and is specifically designed for students who are not confident with Mathematics and statistics. The centre aims to help students registered on any course through a series of events, resources and information.   These include automated Excel calculators for budgeting and workshops targeted at mathematical tests as part of interview / employer processes.

Section 4: Summary

4.1 Discussion

Public perception of dyscalculia is that this is a relatively 'new' addition to the spectrum of specific learning difficulty however it is clear from a review of the literature that identification of specific difficulty in the area of numerosity has been investigated since the early 1990s. There is robust evidence for hemispheric neurological deficits affecting numerical skills and reasoned arguments for a hierarchical cognitive model for acquisition of mathematical skills. Of particular interest is the emphasis on Mathematics as a language system, and how this might affect mathematical understanding and development. However, it also needs to be recognized that acquisition of literacy and numeracy skills is not innate, and that perhaps the development of arithmetical skills is as artificial as learning to read, which may be problematic for some individuals where the brain 'evolved for more general purposes' McCrone (2002). Behaviourally, there is clear evidence of an inability to visualise numbers and to represent them conceptually.

However from an environmental perspective, consideration needs to be given to the effects of ineffective teaching methods, lack of specialised support, the time constraints of the curriculum and inappropriate assessment tools. Evidence from teachers at primary level indicates that there are issues with mathematical language, and assessment of achievement and identification of difficulty using standard assessment methods. Proposed changes to early years Mathematics programmes, and the delivery of the Mathematics curriculum in secondary schools may address some of the problems in the learning and teaching of Mathematics. Intervention programmes emphasise the need for structured and staged approaches which require individualized long term support. However the shortfall in targeted support for those children experiencing very real difficulties in Mathematics – compared to similar provision for literacy - is inadequate.

Performance in state examinations in Mathematics over the period 2001 – 2008 indicates a lack of fluctuation in E, F and NG results, compared with A to D grades. Additionally, results across the three core curriculum subjects of Mathematics, English and Irish over the five year period 2003 – 2008, suggest a statistical difference which may be reflective of accommodations for dyslexia in terms of the spelling and grammar waiver and exemption from Irish, and an absence of accommodations for underlying Mathematics difficulties.

It is the view of the Royal Irish Academy that:

> "Mathematics is not perceived simply as a service subject to be used in other disciplines and that 'mathematical fluency' is recognised as being particularly useful in a wide range of professions (even when not explicitly required)."

Whilst this is undeniable, is it really the case that a qualification in Mathematics to Leaving Certificate level is necessary for all pupils? What extra dimension does it bring in terms of 'real life' skills? Is there an argument that, at its most basic level, the content of the Foundation paper at Junior Certificate level is sufficient for most people to function competently (mathematically) in everyday life?

The Junior Certificate 2009 Foundation paper includes examination of mathematical computation (long division, multiplication, square roots, percentages); problem solving (calculating speed, time and distance, interest); statistics (calculating the mean, histograms, constructing and interpreting graphs); geometry (angles and areas) and probability. For those individuals who have no desire to pursue higher level study with a mathematical component, are basic skills in the above areas sufficient for competency in everyday life skills such as managing a household budget and personal finances? Arguably this has been sufficiently demonstrated in the UK system, where Mathematics is compulsory only to GCSE level.

Johnson et al (2008) state that increasingly students transition to college and only discover that they have a specific learning difficulty which was not identified during second level education. Students with dyscalculia may still achieve success in courses with mathematical components with the right support and tutoring. However, reduced funding for supports means that specific, individually tailored intervention is not always available. In addition, such students need to be aware of the implications that an underlying difficulty might have in terms of course and career choice. Whilst there are a number of support strategies for students exhibiting difficulties with Mathematics, students with dyscalculia require structured advice and guidance prior to applying to the CAO, in terms of course content and course choice. Although third level institutions strive to implement support programmes to address difficulties with Mathematics, arguably such initiatives are a top down approach aimed at 'plugging the gap' in mathematical knowledge.

Clearly there are courses where course content contains a mathematically based core component (Psychology, Sociology, Science and Engineering, for example), and thus competency is an expectation. However issues that need to be reflected upon include:

- the relevance of a pass in Mathematics for arts courses which contain no mathematical element, such as English, Classics or History
- in addition to pure dyscalculia, consideration of a co-morbidity of several disorders / conditions affecting acquisition of mathematical skills
- acknowledgement that pupils with particular disabilities such as visual impairment, have unequal access to the Mathematics curriculum

4.2 Further research

It is clear that any further discussion of the implications and incidence of specific difficulty in Mathematics can only take place on the back of more in depth statistical research and analysis. This might include:

- Monitoring of Mathematics performance at primary level based on models of acquisition of numerical concepts suggested by Geary and Butterworth, against the new early years initiative Aistear.

- Monitoring of Mathematics performance based at secondary level measured against the new Project Maths curriculum.

- Implementation of standardised assessment tools which are appropriate for the assessment of specific difficulty in Mathematics, in comparison to the mainstream.

- Identification of students who do not matriculate on the basis of Mathematics results, and their subsequent educational / work history.

- Identification of the number of mature students registered on undergraduate courses who did not matriculate in Mathematics.

- Investigation of the psycho-educational profiles of students in second level education who are struggling with both the Foundation and Ordinary Level curriculum to determine either the presence of dyscalculia, or poor Mathematics skills as a result of environmental influences.

- Pilot study using the Neuropsychological Test Battery for Number Processing and Calculation in Children to determine incidence of mathematical difficulty in primary school children.

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APPENDICES

APPENDIX 1

Leaving Certificate Statistics 2001 – 2009

Foundation Level

Year	Total number of students	A grade	% of total students	B grade	% of total students	C grade	% of total students	D grade	% of total students	E grade	% of total students	F grade	% of total students	NG	% of total students	Total number of students who failed to matriculate
2001	5227	412	7.8	1662	31.8	1741	33.3	1037	19.8	270	5.2	95	1.8	10	0.2	**375**
2002	5296	480	9.0	1678	25.2	1733	32.7	1028	19.4	260	4.9	103	1.9	14	0.3	**377**
2003	5702	696	12.2	1990	34.9	1739	30.4	952	16.6	245	4.3	70	1.2	10	0.2	**325**
2004	5832	580	10.0	1946	33.4	1863	32.0	1062	18.2	286	4.9	84	1.4	11	0.2	**381**
2005	5562	419	7.5	1733	31.1	1864	33.5	1115	20.0	319	5.7	102	1.8	10	0.2	**431**
2006	5104	400	7.9	1565	30.6	1775	34.7	1027	20.1	247	4.8	84	1.6	6	0.1	**337**
2007	5,580	545	9.7	1,908	34.2	1,742	31.3	1,008	18.1	252	4.5	106	1.9	19	0.3	**377**
2008	5,803	569	9.8	2,010	34.6	1,869	32.2	1,020	17.6	216	3.7	107	1.8	12	0.2	**335**
2009	6,212		10.9		36.0		30.9		17.1		3.7		1.2		0.2	
																2938

Ordinary Level

Year	Total number of students	A grade	% of total students	B grade	% of total students	C grade	% of total students	D grade	% of total students	E grade	% of total students	F grade	% of total students	NG	% of total students	Total number of students who failed to matriculate
2001	39984	5656	14.1	9974	24.9	9219	23.1	8515	21.3	4062	10.2	2228	5.6	330	0.8	**6620**
2002	38932	5281	13.6	9494	24.4	9575	25.0	8967	23.0	3675	9.4	1713	4.4	227	0.6	**5615**
2003	39101	4281	10.9	10384	26.5	10435	27.3	9520	24.4	3164	8.1	1198	3.1	119	0.3	**4481**
2004	37794	5937	15.7	10845	27.8	9390	24.9	7300	19.3	2893	7.7	1239	3.3	190	0.5	**4332**
2005	36773	4886	13.3	10001	27.3	9596	26.1	7872	21.4	2946	8.0	1290	3.5	182	0.5	**4418**
2006	35113	4018	11.4	8599	27.1	9774	27.2	7978	22.7	2872	8.3	1087	3.1	106	0.3	**4065**
2007	35077	4894	13.9	9738	27.7	9251	26.3	7137	20.4	2765	7.8	1143	3.3	147	0.4	**4055**
2008	35808	4483	12.5	10104	28.2	9507	26.6	7373	20.6	2857	8.0	1317	3.7	167	0.5	**4341**
2009	37273		12.7		27.2		27.4		22.3		7.5		2.5		0.3	
TOTAL																**37927**

Higher Level

Year	Total number of students	A grade	% of total students	B grade	% of total students	C grade	% of total students	D grade	% of total students	E grade	% of total students	F grade	% of total students	NG	% of total students	Total number of students who failed to matriculate
2001	9938	2099	21.2	2158	21.7	1886	21	1525	19.2	312	3.1	81	0.8	14	0.1	**407**
2002	9430	1245	13.2	2666	28.3	3154	33.5	1373	20.7	318	3.4	85	0.9	14	0.1	**417**
2003	9453	1257	13.3	2842	30.1	3106	32.9	1843	19.5	334	3.5	59	0.6	12	0.1	**405**
2004	9426	1534	16.2	2823	29.9	2940	31.2	1736	18.4	300	3.2	83	0.9	10	0.1	**393**
2005	9843	1525	15.5	3129	31.9	3029	30.7	1739	17.7	327	3.3	83	0.8	11	0.1	**421**
2006	9018	1280	14.2	3122	34.6	3014	33.4	1307	14.5	222	2.5	63	0.7	10	0.1	**295**
2007	8,388	1,287	15.4	2,836	33.8	2,595	30.9	1,344	16.0	253	3.0	60	0.7	13	0.2	**326**
2008	8,510	1,239	14.6	2,612	30.7	2,792	35.0	1,494	17.5	299	3.5	61	0.7	13	0.2	**373**
2009	8,420		15.0		33.1		32.5		16.0							

3037

Comparison by level

	2001	2002	2003	2004	2005	2006	2007	2008
A	8167	7006	6234	8051	6830	5698	6726	6291
B	13794	13838	15216	15614	14863	13293	14482	14726
C	12846	14462	15280	14193	14489	14563	13588	14168
D	11077	11368	12315	10098	10726	10312	9489	9887
E	4644	4253	3743	3479	3592	3341	3270	3372
F	2404	1901	1327	1406	1475	1234	1309	1485
NG	354	255	141	211	203	122	179	192
Total number of pupils who failed to matriculate on Mathematics	**7402**	**6409**	**5211**	**5096**	**5270**	**4697**	**4758**	**5049**

Comparison by year

		2001	2002	2003	2004	2005	2006	2007	2008
Higher	A	2099	1245	1257	1534	1525	1280	1,287	1,239
	B	2158	2666	2842	2823	3129	3122	2,836	2,612
	C	1886	3154	3106	2940	3029	3014	2,595	2,792
	D	1525	1373	1843	1736	1739	1307	1,344	1,494
	E	312	318	334	300	327	222	253	299
	F	81	85	59	83	83	63	60	61
	NG	14	14	12	10	11	10	13	13
Non-matriculation		**407**	**417**	**405**	**393**	**421**	**295**	**326**	**373**

Ordinary	A	5656	5281	4281	5937	4886	4018	4894	4483
	B	9974	9494	10384	10845	10001	8599	9738	10104
	C	9219	9575	10435	9390	9596	9774	9251	9507
	D	8515	8967	9520	7300	7872	7978	7137	7373
	E	4062	3675	3164	2893	2946	2872	2765	2857
	F	2228	1713	1198	1239	1290	1087	1143	1317
	NG	330	227	119	190	182	106	147	167
Non-matriculation		**6620**	**5615**	**4481**	**4322**	**4418**	**4065**	**4055**	**4341**

Foundation	A	412	480	696	580	419	400	545	569
	B	1662	1678	1990	1946	1733	1565	1,908	2,010
	C	1741	1733	1739	1863	1864	1775	1,742	1,869
	D	1037	1028	952	1062	1115	1027	1,008	1,020
	E	270	260	245	286	319	247	252	216
	F	95	103	70	84	102	84	106	107
	NG	10	14	10	11	10	6	19	12
Non-matriculation		**375**	**377**	**325**	**381**	**431**	**337**	**377**	**335**

	2001	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**
Total number of pupils who failed to matriculate on Mathematics	**7402**	**6409**	**5211**	**5096**	**5270**	**4697**	**4758**	**5049**

TABLE 1
Subtypes of Learning Disabilities in Mathematics

Cognitive and performance features	Neuropsychological features	Genetic features	Developmental features	Relation to RD
Procedural Subtype				
Relatively frequent use of developmentally immature procedures (i.e., the use of procedures that are more commonly used by younger, typically achieving children) Frequent errors in the execution of procedures Poor understanding of the concepts underlying procedural use Difficulties sequencing the multiple steps in complex procedures	Unclear, although some data suggest an association with left hemispheric dysfunction and, in some cases, (especially for sequencing problems) a prefrontal dysfunction	Unclear	Appears, in many cases, to represent a developmental delay (i.e., performance is similar to that of younger, typically achieving children and often improves across age and grade)	Unclear
Semantic Memory Subtype				
Difficulties retrieving mathematical facts, such as answers to simple arithmetic problems For facts that are retrieved, there is a high error rate For arithmetic, retrieval errors are often associates of numbers in the problem (e.g., retrieving 4 to 2 + 3 = ?; 4 is the counting-string associate that follows 2, 3) RTs for correct retrieval are unsystematic	Appears to be associated with left hemispheric dysfunction, possibly the posterior regions for one form of retrieval deficit and the prefrontal regions for another Possible subcortical involvement, such as the basal ganglia	Appears to be a heritable deficit	Appears to represent a developmental difference (i.e., cognitive and performance features differ from those of younger, typically achieving children and do not change substantively across age or grade)	Appears to occur with phonetic forms of RD
Visuospatial Subtype				
Difficulties in spatially representing numerical and other forms of mathematical information and relationships Frequent misinterpretation or misunderstanding of spatially represented information	Appears to be associated with right hemispheric dysfunction, in particular, posterior regions of the right hemisphere, although the parietal cortex of the left hemisphere may be implicated as well	Unclear, although the cognitive and performance features are common with certain genetic disorders (e.g., Turner's syndrome)	Unclear	Does not appear to be related

Note. From "Mathematical Disabilities: Cognitive, Neuropsychological, and Genetic Components," by D. C. Geary, 1993, *Psychological Bulletin, 114*, p. 362. Copyright 1993 by the American Psychological Association. Adapted with permission. RD = reading disabilities; RT = reaction time.

APPENDIX 3

Biological primary quantitative abilities	
Numerosity	The ability to determine accurately the quantity of small sets of items, or events, without counting. Accurate numerosity judgments are typically limited to sets of four or fewer items (from infancy to old age).
Ordinality	A basic understanding of more than and less than, and, later, an understanding that 4 > 3; 3 > 2; and 2 > 1. Early limits of this system are not known, but appear to be limited to quantities of < 5.
Counting	Early in development there appears to be a preverbal counting system that can be used for the enumeration of sets up to 3, perhaps 4, items. With the advent of language and the learning of number words, there appears to be a pan-cultural understanding that serial-ordered number words can be used for counting, measurement, and simple arithmetic.
Simple arithmetic	Early in development there appears to be sensitivity to increases (addition) and decreases (subtraction) in the quantity of small sets. In infancy, this system appears to be limited to the addition or subtraction of items within sets of 2, and gradually improves to include larger sets, although the limits of this system are not currently known.

Adapted from Geary, 2009

Biologically secondary number, counting, and arithmetic competencies	
Number and counting: Mastery of the counting system, gain an understanding of the base-10 system, and learn to translate, or transcode, numbers from one representation to another	Verbal two hundred ten to Arabic 210), counting errors common for teen values (e.g., forgetting the number word) and for decade transitions (e.g., 29 to 30, often misstated as twenty nine, twenty ten). Number transcoding errors (two hundred ten as 20010) are common in primary school children, especially in the first few grades. Learning the base-10 system appears to be the most difficult counting and number concept that primary school children are expected to learn, and many never gain a full understanding of the system.
Arithmetic: computations Basic arithmetic facts and learn computational procedures for solving complex arithmetic problems	With sufficient practice, nearly all academically normal children will memorize most basic arithmetic facts; in some countries, however, the level of practice is not sufficient to result in the memorization of these facts, which, in turn, results in retrieval errors and prolonged use of counting strategies. The ability to solve complex arithmetic problems is facilitated by the memorization of

	basic facts, the memorization of the associated procedures, and an understanding of the base-10 system. The latter is especially important for problems that involve borrowing or carrying (e.g., 457+769) from one column to the next.
Arithmetic: word problems Begin to solve simple word problems	Complexity of the problems they are expected to solve in later grades varies greatly from one nation to the next. The primary source of difficulty in solving these problems is identifying problem type (e.g., comparing two quantities vs. changing the value of one quantity) and translating and integrating the verbal representations into mathematical representations. In secondary school, the complexity of these problems increases greatly and typically involves multi-step problems, whereby two or more verbal representations must be translated and integrated. Without sufficient practice, the translation and integration phases of solving word problems remain a common source of errors, even for college students.

Adapted from Geary, 2009

Dyslexia Screener Teacher/Practitioner Report

GL assessment
the measure of potential

Stanine (ST)

Stanines (short for 'standard nines') are a simplification of the standard age score that divides the SAS into nine broader bands. They show how a student performed on a test in comparison with the national sample, with 9 being the highest score and 1 being the lowest.

The broad nature of stanines minimises the over-interpretation of small, insignificant differences among test scores. Stanines are therefore particularly useful in reporting test information to pupils and to parents, as they are relatively easy to understand and interpret.

Standard Age Score (SAS)

The standard age score is based on the underlying raw score and enables you to compare your own pupils with a larger, nationally representative sample of pupils of the same age that have taken the test prior to publication.

The national average standardised score is 100, irrespective of the difficulty of the test, and so it is easy to see whether a pupil is above or below the national average.

National Percentile Rank (NPR)

The national percentile rank indicates the percentage of pupils in the national sample who obtain a standard age score at or below a particular score. For example, a pupil with a standard age score of 108 has a national percentile rank (NPR) of 70: he or she has performed as well as, or better than, 70 per cent of pupils of his or her age group.

An NPR of 50 is average for an age group.

The Dyslexia Index

The Dyslexia Index is an overall indicator of the extent to which a test taker's profile of results matches that which is commonly found for people with dyslexia.

The Index is calculated by a mathematical formula using all six individual sub-test raw scores plus two other scores, 'expected reading' and 'expected spelling', which are calculated from the combined ability (Missing Pieces and Vocabulary) score. The values range from A, which signifies no evidence of a dyslexic profile, to E, which signifies evidence of a severe dyslexic profile. Most dyslexic individuals fall into category C.

Letter code	Description
A	No signs of dyslexia
B	Few signs of dyslexia
C	Mild dyslexia
D	Moderate dyslexia
E	Severe dyslexia

The Dyslexia Index value 'A' generally means that no evidence of dyslexic tendencies has been found and no further action is necessary as a consequence. However, there are some profiles yielding an 'A' that suggest the need for follow-up and these are noted in the individual and group reports.

Flat low profile

Students who produce uniformly low scores need further investigation into the nature of their difficulties, to find out if they really have general cognitive difficulties or if their current low performance stems from emotional or motivational roots.

Flat high profile

Students who produce uniformly high scores need highlighting in case their educational potential has not yet been recognised.

Reverse Dyslexia

A few students may yield an anomalous 'overachievement' profile, in which they appear to be performing better in literacy than their ability level would indicate was likely. These cases need further investigation, to identify why their ability scores were unusually low, given their educational achievement.

Low attainment

Students who do not produce a dyslexic profile but nevertheless show low attainment in literacy need highlighting, as they might not be able to access an ability-appropriate curriculum without support.

The Sub-tests

The six-test model is organised as follows:

	Type	Name	Description
1	Ability	Missing Pieces	Non-verbal reasoning
2	Diagnostic	Word Sounds	Phonological processing
3	Attainment	Spelling	Letter recognition, word segmentation and proofing
4	Diagnostic	Visual Search	Perceptual speed
5	Attainment	Reading	Word recognition and comprehension
6	Ability	Vocabulary	Verbal comprehension

Ability tests

The ability tests address different aspects of general problem solving ability.

Missing Pieces assesses how well a learner can recognise similarities, differences and relationships in shapes and designs.

Vocabulary assesses the learner's knowledge of word meanings.

Diagnostic tests

The diagnostic tests sample the information-processing efficiency in two domains - perceptual speed and the processing of the sounds of words.

Word Sounds assesses how well a learner can identify individual sounds from within words.

Visual Search assesses the speed at which a learner can process simple visual information.

Attainment tests

The attainment tests are of reading and spelling, particularly word-level processes.

Reading assesses how well a learner can recognise spoken words and select the correct word to complete sentences.

Spelling assesses how well a learner can select letters, correctly spelt words and parts of words.

APPENDIX 4

Sharma, M. (1989) *How Children Learn Mathematics*: Professor Mahesh Sharma, in interview with Bill Domoney. London, England: Oxford Polytechnic, School of Education. 90 min. Educational Methods Unit. Videocassette.

(1) A teacher must first determine each student's cognitive level (low--high) of awareness of the knowledge in question, and the strategies he brings to the Mathematics task. Low functioning children have not mastered number preservation and are dependent on fingers and objects for counting. Findings dictate which activities, materials, and pedagogy are used (Sharma 1989).

(2) The teacher must understand that each student processes math differently, and this unique learning style affects processing, application, and understanding. Quantitative learners like to deal exclusively with entities that have determinable magnitudes. They prefer the procedural sequences of math. They methodologically break down problems, solve them, and then assemble the component solutions to successfully resolve a larger problem. They prefer to reason deductively, from the general principle to a particular instance (Sharma 1990, 22).

Quantitative students learn best with a highly structured, continuous linear focus, and prefer one standardized way of problem solving. Introductions of new approaches are threatening and uncomfortable- an irritating distraction from their pragmatic focus. Use hands-on materials, where appropriate (Sharma 1989).

Qualitative learners approach math tasks holistically and intuitively, with a natural understanding that is not the result of conscious attention or reasoning. Based on descriptions and characteristics of an element's qualities they define or restrict the role of math elements. They draw parallels and associations between familiar situations and the task at hand. Most of their math knowledge is gained by seeing interrelationships between procedures and concepts.

Qualitative learners focus on recognizable patterns and visual/spatial aspects of information, and do best with applications. They are social, talkative learners who reason by verbalizing through questions, associations, and concrete examples. They have difficulty with sequences and elementary math (Sharma 1990, 22).

- Qualitative learners need continuous visual-spatial materials. They can successfully handle the simultaneous consideration of multiple problem solving strategies and a discontinuous teaching style of demonstration and explanation, stopping for discussion, and resumption of teaching (Sharma 1989); whereas this style may agitate the qualitative learner who resents disruptions to linear thought.

- For each student, the teacher must assess the existence and extent of math-readiness skills. Non-mathematical in nature, mastery of these seven skills is essential for learning the most basic math concepts (Sharma 1989).

  The seven prerequisite math skills are:

  (1)The ability to follow sequential directions;

  (2) A keen sense of directionality, of one's position in space, and of spatial orientation and organization;

  (3) Pattern recognition and extension;

  (4) Visualization- key for qualitative students- is the ability to conjure up and manipulate mental images;

  (5) Estimation- the ability to form a reasonable educated guess about size, amount, number, and magnitude;

  (6) Deductive reasoning- the ability to reason from the general principle to a particular instance;

  (7) Inductive reasoning- natural understanding that is not the result of conscious attention or reason, easily seeing patterns in situations, and interrelationships between procedures and concepts (Sharma 1989).

  (4) Teachers must teach math as a second language that is

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  exclusively bound to the symbolic representation of ideas. The syntax, terminology, and translation from English to math language, and math to English must be directly and deliberately taught.

- Students must be taught the relationship to the whole of each word in the term, just as students of English are taught that "boy" is a noun that denotes a particular class, while "tall," an adjective, modifies or restricts an element (boy) of a particular class (all boys). Adding another adjective, "handsome," further narrows or defines the boy's place in the class of all boys. At all times, concepts should be graphically illustrated.

  The concepts of "least common multiple" and "tall handsome boys" look like this (Sharma 1989):

  Figure 1: Illustrating linguistic concepts

  All boys Tall boys Handsome tall boys

  All multiples Common multiples Least common multiples

  The language of Mathematics has a rigid syntax that is easily misinterpreted during translation, and is especially problematic for students with directional and sequential confusion. For example, "86 take away 5," may be written correctly in the exact order stated: 86-5.

When the problem is presented as "subtract 5 from 86," the student may follow the presented order and write 5-86. Therefore, it is essential that students are taught to identify and correctly translate math syntax (Sharma 1989). The dynamics of language translation must be deliberately and directly taught.

Two distinct skills are required. (1) Students are usually taught to translate English expressions into mathematical expressions. (2) But first, they should be taught to translate mathematical language into English expression. Instead of story problems, Sharma advocates giving the child mathematical expressions to be translated into or exemplified by stories in English.

- Without becoming overwhelmed with the prospect of addressing each child's needs individually, the continuum can be easily covered by following Sharma's researched and proven method. It is outlined below. After determining that students have all prerequisite skills and levels of cognitive understanding, introduce new concepts in the following sequence:

  (A) Inductive Approach for Qualitative Learners:

  (1) Explain the linguistic aspects of the concept.

  (2) Introduce the general principle, truth, or law that other truths hinge on.

  (3) Let the students use investigations with concrete materials to discover proofs of these truths.

  (4) Give many specific examples of these truths using the concrete materials.

  (5) Have students talk about their discoveries about how the concept works.

  (6) Then show how these individual experiences can be integrated into a general principle or rule that pertains equally to each example.

  (B) Deductive Approach for Quantitative Learners: Next, use the typical deductive classroom approach.

  (7) Reemphasize the general law, rule, principle, or truth that other mathematical truths hinge on.

  (8) Show how several specific examples obey the general rule.

  (9) Have students state the rule and offer specific examples that obey it.

  (10) Have students explain the linguistic elements of the concept (Sharma 1989).

- Proven programs of prevention, systematic evaluation, identification of learning difficulties, early intervention, and remediation in Mathematics must be implemented *immediately* to reverse dismal achievement statistics and to secure better educational and economic outcomes for America's students.

APPENDIX 5

Mathematics software programmes

Mathematics Pad

Compatibility/Requirements:
Macintosh system 7 or higher, 4 MB RAM
Grades: Primary / secondary
Price: $79.95 (single copy) $50.00 each (multi-user; 25 users or more)
MathematicsPad is the ideal solution for students who:
* Need help organizing Mathematics problems
* Have difficulty doing pencil and paper Mathematics
* Have vision problems that require large-size print, high-contrast background colors or speech feedback.

Mathematics Shop Series
Compatibility/Requirements:
PC/DOS or Macintosh-Bilingual version in DOS available
Age range: Primary:
Price:$29.95 for each shop

Helping customers in the real world environment of a shopping mart, students gain a sense of how Mathematics is applied in everyday life. Whether they are calculating the area of carpet, loading a soda truck, or boxing cartons of eggs, students will need to make maximum use of their Mathematics skills. There are four shops for the students to work in:

Mathematics Shop includes addition, subtraction, multiplication, division, fractions, decimals, ratios and more.

Mathematics Shop Spotlight: Weights and Measures focuses on pounds and ounces; feet and inches; cups,

Pints, quarts and gallons; days and weeks; and more.

Mathematics Shop Spotlight: Fractions & Decimals teaches students how to add and subtract fractions, multiply fractions by whole numbers, convert decimals to fractions, and add decimals.

Algebra Shop covers factoring, squares/square roots, cubes/cube roots, number series and more.

Mathematics Trek 7,8,9

Compatibility/Requirements:
Macintosh or Windows
Age range: Primary
Price: $84.95 each module

The modules in this product cover the following topics:
* Algebra
* Fractions
* Geometry
* Graphing
* Integers and Percents
* Whole Numbers and Decimals
Multilingual program includes English, Spanish, and French versions.

Mathematics Trek 10,11,12

Compatibility/Requirements:
Macintosh or Windows
Age range: Secondary
Price: $59.95 each module
Covers the following topics.
* Factoring
* Systems of Equations
* Statistics, Probability
* Coordinate Geometry
* Transformational Geometry
* Second Degree Relations
* Quadratic Functions
* Mathematical Tools
* Student Tracking System

The Mathematics Tools module includes a charting tool, spreadsheet, a Mathematics word processor, probability tools, and algebra tiles. A Student Tracking System module is also available.

This program is at a high level of difficulty.

Mathematicscad 5.0/6.0

Compatibility/Requirements:

386 or higher IBM or compatible PC, 4 MB RAM, 15MB free disk space, MS Windows 3.1

Ages: College

Price: $129.95 - $349.95

Mathematicscad is a computer program, which helps solve mathematical calculations.

This program helps turn the computer screen into a worksheet. The user types the equation onto the screen and Mathematicscad solves it.

The user can type the formula or choose symbols from Mathematicscad's palette. This program can solve equations from algebra to calculus, and text can be added to create a document. This program runs in Windows only, not in DOS.

Mathematics Home Work

Compatibility/Requirements: Macintosh family only

Age range: All

Price: $29

Mathematics Home Work is especially useful for students who need to use a computer for written work. It is a template for creating, solving, and printing Mathematics problems in all basic operations, including fractions and decimals.

It is configured to proofread calculations automatically as they are entered and to identify where in the problem an error occurred, without solving the error for the user.

A separate column is provided for visual clarity.

This product allows only two font sizes (9 and 12 point), which might be problematical for low-vision students.

Mathematicspert Algebra Assistant

Compatibility/Requirements: Windows 3.1, Win95, Windows NT

386 or higher (speed), 6 MB (hard drive space), 8 MB minimum (memory), 256-color VGA 1X (CD ROM speed), Windows compatible sound stereo, Windows compatible mouse

Age range: High School and College

Price: $95.00

Mathematicspert Algebra Assistant is a professional Mathematics program that utilizes active intelligence to solve any course-level problem in Algebra I and II. The system is unique in its ability to analyze any problem and display the solution in a correct series of steps. Its remarkable mathematical power is combined with an easy-to-use graphical interface that guides students through problems in the same step-by-step manner as taught in class. The student is placed in full control of the operational strategy, while the computer takes care of the mathematical details and provides protection from trivial mistakes. The program offers hints, assistance and complete step-by-step solutions when requested, and always displays the mathematical justification for each operation.

Algebra Assistant builds experience and proficiency in the strategy of problem solving-the essence of mathematical mastery. In addition to its unmatched capabilities for solving any problem, the program contains over 2, 500 typical textbook exercises organized by standard algebra topics. Students can go directly to any topic where extra help and practice is needed, including factoring, linear equations and inequalities, quadratic equations, fractions, exponents, roots and more.

Operation Neptune (CD)

Compatibility/Requirements: Macintosh or Windows

Age range: Primary

Price: $57.95

This product helps prepare students for algebra.

Students read and interpret real-world graphs, charts, maps, and other tools as they navigate the submarine Neptune.

Students develop Mathematics skills with whole numbers, fractions, decimals and percentages, and practice using measurement concepts, including time, distance, speed, angles, area, and volume.

Theorist

Compatibility/Requirements: Any Macintosh

Age range: Third level

Price: $299.00

This program can help the student take on any challenges in the undergraduate Mathematics curriculum.

It is one of the first symbolic-Mathematics programs for the Macintosh and is geared for a student taking a freshman calculus class in college.

Its strength is its interface: the student can enter an equation with just a few clicks, graph or simplify the equation with a single click, and tinker with it endlessly. The interface encourages exploration, and the tutorial makes exploration easy and entertaining.

The Trigonometry Explorer (CD)

Compatibility/Requirements: Macintosh or Windows, 4 MB RAM

Age range: Secondary

Price: $129.95

Students discover applications of trigonometry for science and social studies and see how trigonometry relates to algebra and geometry as they measure the distance to the stars; experiment with sound, light, and radio waves; explore bridge construction with triangles; and travel back in time to discover how Eratosthenes first measured the circumference of the earth.

The animated lessons are easy to follow and are presented in a logical, concept-building format. Within the lessons, students are encouraged to explore different aspects of trigonometry and apply what they have learned to randomly generated practice problems.

After each lesson real-world applications with graphic animations and QuickTime movies reinforce students' learning and suggest new avenues of exploration.

APPENDIX 6

Chapter 5 Actions

Having regard to our research to date Engineers Ireland commit to the following in terms of the assistance and support we can give to the better education of Mathematics and Science at Second Level:

Action 1
That Engineers Ireland as the authoritative voice of Irish Engineering and as a leading professional group in the ‘knowledge economy’ seek a voice in the NCCA in the future direction of curriculum change in subjects relevant to our profession at Second Level i.e. Mathematics, Applied Mathematics, Physics and Chemistry and in implementation groups with this purpose.

Action 2
That Engineers Ireland offer award incentives to teachers to retrain and up-skill to meet the challenges of new syllabi in Mathematics and Science subjects. These incentives could take the form of sponsored scholarship schemes or alternative award schemes for ‘outstanding merit’ including the BT Young Scientist Awards.

Action 3
Due to the current downturn in the construction industry, advantage could be taken of retraining engineers as Mathematics and Science teachers. This is subject to them acquiring an acceptable post graduate degree or diploma qualification in Education similar to the new NUIG, UL, DCU and NUIM Mathematics and Education degrees and support courses in DIT, CIT & WIT. Engineers Ireland must encourage and promote this development with the Teaching Council and NCCA and seek possible tax breaks for the retraining of personnel.

Action 4
Engineers Ireland need to awaken greater interest in Project Mathematics/Science at both Primary and Second Level by better integration into the STEPS Programme to ensure more holistic and integrated learning towards Engineering and Science subjects and with particular regard to Transition Year teaching and students. The STEPS Programme should be re-examined and strengthened to help fulfill the Engineers Ireland recommendations in this report.

Action 5
Engineers Ireland should lead a greater use of the power of ICT to contextualise the teaching of Mathematics and Science at Primary and Second Level.

Action 6
Engineers Ireland should set-up on our new website a Wiki-Solution web page to assist students with problem solving in Mathematics and Applied Mathematics and link with other relevant sites.

Action 7
Engineers Ireland should consider setting up a new Education Division to attract Third Level professors and lecturers in Mathematics, Science and Engineering to join and participate in greater numbers. We should also include Primary and Second Level teachers at meetings on a regional level to aid improved communication between teachers and engineers on a professional level. The essential continuity links between Primary and Second Level need to be emphasised in these regional 'conversations'.

Action 8
There needs to be more formal links between Engineers Ireland and Women in Technology and Science (WITS) to ensure greater gender integration in Mathematics, Science and Technology courses.

Action 9
The rising failure rates at Ordinary Level Leaving Certificate Mathematics must be urgently examined as it will seriously impact on the future standard of technicians (Level 6/7) in Ireland.

Action 10
There is a significant opportunity for interactions in Transition Year by Engineers Ireland. We must make it more practical with topical projects within the Project Mathematics, Science and Engineering fields. There are great future opportunities for Engineers Ireland to link with Second Level schools, Teacher Associations/Unions and Industry to assist further the development of the 'smart economy'.