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# MEASURING THE EFFECTIVENESS OF A MATHS LEARNING SUPPORT CENTRE - THE DCU EXPERIENCE

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The Maths Learning Centre (MLC) in DCU opened in February 2004 and completed its first full academic year of operation in 2004-05. Its aim is to provide additional assistance to all undergraduate DCU students taking a mathematics course as part of their degree programme. The MLC has the particular remit of addressing student retention by assisting those first year students who struggle with their mathematics module. To address the effectiveness of the MLC we studied (i) usage statistics, (ii) student feedback on the MLC, including interviews and (iii) the pass rates of students who were judged to be at risk of failing their first-year maths module. Under (iii), this cohort was sub-divided into students who attended the MLC and those who did not. By comparing the pass rates of these two groups, we argue that the MLC has made a positive contribution to student retention. This is among the points made in an ongoing proposal for the permanent establishment of a Maths Learning Centre in DCU.

## Support offered by the MLC

To help support mathematics learning in DCU, the MLC offers:

- Individual consultation
- Diagnostic testing
- Front-end refresher sessions
- Exam revision sessions
- Support for special groups (Access students, mature students)
- Computer based learning
- Study area in CG10a
- Access to course material and texts
- Research and reporting

Use of the MLC is free.

## Student Cohorts

For the purposes of measuring the MLC's effectiveness, students from the large first-year service maths modules are defined as target students.

During the summer of 2004, a diagnostic test for the MLC was developed. The test contains 15 multiple choice questions on basic mathematical skills. The questions are roughly at ordinary level leaving certificate standard, and are based on what staff teaching first year students judge to be the 'atoms' of mathematical calculations. The questions are on percentages and fractions, numerical and algebraic manipulation and solving (linear and quadratic) equations. Students who scored 6 or less in the test were categorised by the MLC as at-risk students and advised to visit the MLC as early and as frequently as possible. However, as many of the students who scored 7 or 8 in

this test failed their maths module, the at-risk bar was raised to 8 for 2005/06. This explains the increase in the number of at-risk students that year (see Table 3).

## Usage Numbers

Given the MLC's remit to aid retention, particular attention was paid to the attendance by target and at-risk students. These are tabulated below.

	2003/04	2004/05	2005/06
Number of Visits	507	1017	1701
Number of Students	156	393	476

**Table 1: Usage Numbers for All Students**

	2004/05	2005/06
Number of Target Students	811	826
Number of Target Students Who used the MLC	260	408
Number of Target Students Who used the MLC	32%	49%

**Table 2: Usage Numbers for Target Students**

	2004/05	2005/06
Number of At-risk Students	80	161
Number of At-risk Students Who used the MLC	41	95
Number of At-risk Students Who used the MLC	51%	59%

**Table 3: Usage Numbers for At-risk Students**

## Student Feedback

A detailed questionnaire was developed by the MLC's staff and was handed out to target students at the end of the last two academic years. Statements were presented with which students had to indicate their level of agreement. This approach allows

easier display of the data collected. 161 students who completed the questionnaire had visited the MLC. Here, we only consider the responses of these students to a selection of the statements. The results are shown in Table 4 on the next page.

At the end of the 2004/05 academic year a number of students who visited the MLC regularly were asked to partake in structured interviews with the manager of the MLC. The interviews took place during May 2005 in the MLC. This was repeated in May 2006. The interviews indicated a very positive opinion of the MLC and its operation. Students regularly emphasised the one-to-one tuition as a positive aspect of the MLC.

‘The support was very individual and it was good. It was one-to-one so if I didn’t understand something it was at my own pace.’

‘..you got individual attention and the tutors knew that not everybody had done honours maths for the Leaving.’

Many also liked that they were given time to work on the problems themselves and they could then call on a tutor.

‘I’d come in and ask then questions and then get the help. Yet it wasn’t just giving the answers to me, I was able to work through them myself and then if I came to a stage where I couldn’t continue then I would have been helped’

The Centre’s relaxed atmosphere was also praised.

‘It was very relaxed when I came in...I didn’t feel like I was stupid or anything.’

‘I liked the fact it wasn’t very formal. You get help as you need it. You can work on your own then for awhile.’

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
The tutors in the MLC are approachable and patient	58%	37%	3%	1%	0%
Overall the tutors in the MLC perform well	53%	40%	6%	1%	0%
I have benefited from using the MLC	52%	39%	9%	0%	0%
The MLC contributes	45%	38%	13%	3%	1%

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significantly  
to my maths  
learning

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**Table 4: Results from the Students Questionnaire**

### **Student Retention**

As previously mentioned, the MLC has a particular remit in respect to student retention, and has made a positive contribution in this regard. The following tables show examination success among target and at-risk students, contrasting the performance of those students who did and did not attend the MLC.

	2004/05	2005/06
Target students who used the MLC	79%	74%
Target students who didn't use the MLC	76%	68%

**Table 5: Pass rates for Target Students**

	2004/05	2005/06
At-risk students who used the MLC	53%	60%
At-risk students who didn't use the MLC	25%	49%

**Table 6: Pass rates for At-risk Students**

In 2004/05, there were 80 at-risk students. Of these 41 attended the Centre. 22 of these passed their maths module. Applying the pass rate of 25% among non-attendees to 41 gives us 10 students. It is reasonable to assume that the Centre contributed to the exam success of 12 students. Applying this process to the 2005/06 figures gives us 10 students. It can therefore be argued that the Centre has contributed 22 students passing their first-year maths module over the last two years, with the consequential developments (financial and other) accruing to DCU.

### **Issues in running the MLC**

- We need more mechanisms to identify at-risk students;
- Getting at-risk students to visit the Centre early and often;
- Students arriving unprepared;
- Students expecting us to do all their work;
- Students only using the Centre and engaging the course material before a class test;
- Students forgetting material they have previously covered in the Centre.

To deal with some of these issues, we will introduce the following ground rules:

- You must bring the necessary material to the Centre: pens, paper, calculator, lecture notes, tutorial sheets, text books;

- The Centre is not to be used as a substitute for lectures and tutorials;
- If you are stuck on a particular question, you must bring your attempts at this problem to the Centre;
- If you are having difficulties in general with maths, you should attend the Centre regularly and as early in the academic year as possible;
- If you come to the Centre with a mathematical question arising from either project work or a non-mathematics module, the Centre will endeavour to provide assistance.

## **Conclusion**

To date, the MLC at DCU has been successful in answering its remit of providing a third leg for mathematics learning at DCU. This is supported by usage statistics and student feedback. Furthermore, there is strong (but admittedly not conclusive) evidence that the Centre has contributed positively to student retention. This is one of several issues that will be the subject of research at the Centre in the coming years. Others include the effectiveness of the different learning tools provided by the centre – one-to-one help, software etc – and the influence of the Centre on students' learning styles.

The data presented here are important not only for the crucial task of assessing the effectiveness of the Centre and its different activities, but will also be useful in influencing budget holders and policy makers to continue and mainstream the funding of the MLC at DCU. There has already been some success in this regard, in that the provision of such data to the University has been a major factor in the continuation of the Centre's funding up to March 2007.

# WHAT IS RELEVANT KNOWLEDGE FOR PROBLEM SOLVING IN PHYSICS?

Gunnar Friege, Institute for Science Education at the University of Kiel

It is a much-lamented fact that students often do not succeed in applying knowledge which they have acquired in lessons to solve problems in school or everyday life contexts. This circumstance seems to apply especially to science lessons. Although the problem solving literature is quite extensive, less is known about the characteristics of 'relevant knowledge' for problem solving. Based on empirical findings and theoretical considerations related to the field of expertise research, the results of an investigation about the importance of different 'types' and 'qualities' of knowledge in relation to problem solving in physics are discussed. Types of knowledge are e.g. situational knowledge or procedural knowledge. Aspects of the quality of knowledge are e.g. the level of automation or the level of abstraction. Furthermore, the study asked how knowledge of good and weaker problem solvers (experts and novices) in the subject physics differ from one another. Consequences of these results for improving science education are discussed.

## **Introduction:**

The field of expertise research became an extensive and fast growing branch of cognitive psychology in the last three decades (e.g. Chi et al., 1988; Gruber & Ziegler, 1996). On a general level expertise research tries to understand, how people (experts), who continuously achieve extraordinary results in a specific field differ from such (novices), who do not. There is no commonly accepted definition of the terms expert and novice, but there is a consensus that expertise is a domain-specific and lasting phenomenon (Posner, 1988) and that it presupposes extensive practice. In expertise research different fields like chess, betting at horseraces, physics or basketball and many more were examined. The standard method is to compare experts and novices in a specific field.

Research on science expertise has a natural affinity to science education because novices try to become experts by learning. Therefore studies on differences between both groups may give hints for a successful development of expertise in the natural sciences. Also, both, expertise research and science education, put stress on domain-specific knowledge. In the last years the pedagogical importance of research on expertise has been enhanced because, firstly not only the very first beginners and the high grade experts were contrasted, but also attention was given to the levels between the extremes and secondly the learning processes of promising novices were studied directly, especially the learning by self-explaining. Research on expertise has revealed various differences between novices and experts. Three important findings are that good and poor problem solvers differ with respect to (1) the structure of their domain-specific knowledge, (2) how they mentally represent problems and (3) the way they learn when dealing with problems. There are, however, some research deficits in this still very young field of research which cause limited instructional implications especially for improving teaching in schools to help novices to become experts more effectively.

Over many years an interdisciplinary expertise research group consisting of physics and biology educators, pedagogues and psychologists were working together within the Institute of Science Education (IPN) at the university of Kiel. The main working fields of the groups were 1. expert and novice learning in physics and biology and 2.

knowledge-centered problem solving in physics. For example, the generation of self-explanation while working with worked-out-examples and while solving problems have been investigated (Lind, Friege, Sandmann & Kleinschmidt, 2004; Lind, Friege, & Sandmann, 2005); a model of knowledge-centered problem solving has been examined (Friege, 2001) and the connection between intelligence as a domain independent construct and the domain-specific problem solving ability in physics (Friege & Lind, 2003). This paper is about the empirical examination of types and qualities of knowledge and their relations to problem solving in physics. A much more detailed account of this research can be found in a recently published paper (Friege & Lind, 2006).

### **Types and qualities of knowledge:**

It is a much-lamented fact that students often do not succeed in applying knowledge which they have acquired in lessons, to solve problems given in school or in everyday life contexts. The importance of different kinds of knowledge in relation to problem solving will be discussed here. Furthermore, the question is seized, in which way the knowledge of good and weaker problem-solvers (experts and novices) in the subject physics differ from one another.

The abilities of “experts” imply both competence and knowledge. Mostly, competence is taken as the criterion of expertise, i.e. managing problem situations which are typical for the considered domain. In well-structured domains such as physics, the problem situations which students usually have to cope with are mostly problems with a unique solution. Expertise at school level may then be equated with the competence to solve physical problems. If knowledge is considered to be the necessary precondition of competence, it seems reasonable to look at knowledge in order to find an explanation of expertise. The central questions of the different approaches to develop a theory of expertise are the following: How can the big domain-specific memory performance of experts be explained? In which ways does the knowledge of experts differ from that of novices? There is a consensus that the essential differences lie in the type of knowledge and its storage. The configuration of the cognitive apparatus is generally considered equal for experts and novices.

Theoretical considerations and cognitive models in this research field are based predominantly on learning experiments and memory experiments with simple structured materials. Less is known about how knowledge which is relevant for problem solving should be organised in a domain as rich in content as physics. Almost no study so far has attempted to discuss the characteristics of “relevant knowledge” (Kintsch & Ericsson, 1996). There are many studies which single out specific features of knowledge with regard to its importance for problem solving. Furthermore, there are attempts to put these results in order for classifying types of knowledge (de Jong & Ferguson-Hessler, 1996; Krems, 1994). It is possible to award the status of theoretically founded taxonomies to these classifications because they were set up against the background of the theories of expertise. These taxonomies are useful to define the general frame in empirical studies of a certain domain.

De Jong & Ferguson-Hessler (1996) distinguish between various ‘types of knowledge’ and different aspects of ‘quality of knowledge’ which can occur in all types of knowledge. ‘Types of knowledge’ are i.e. situational knowledge (knowledge about typical problem situations), conceptual knowledge (facts, concepts, principles



of a domain), procedural knowledge (knowledge about actions which are important for problem solving). Aspects of ‘quality of knowledge’ are i.e. hierarchical organisation (superficial vs. deeply embedded), inner structure (isolated knowledge elements vs. well structured, interlinked knowledge), level of automation (declarative vs. compiled, in terms of Anderson, 1987), or level of abstraction (colloquial vs. formal, in terms of Krems, 1994). Of course, ‘types’ and ‘qualities’ do not exist independently of each other. Often declarative knowledge turns out to be of conceptual nature and compiled knowledge contains procedural elements (Anderson, 1987; Singley & Anderson, 1989). A very specific kind of knowledge is the problem scheme knowledge (Van Lehn, 1989) that is typical for experts. It is a specific combination of ‘types’ and ‘qualities’ of knowledge and consists of very profound and interlinked situational, procedural and conceptual knowledge of high quality.

### **Research Aims and research questions:**

The general aims of the empirical investigations described in this report were a) the examination of different ‘types of knowledge’ and different features of the ‘quality of knowledge’ with respect to their effect on problem solving of experts and novices, b) the investigation of experts and novices of similar performance levels in one study, i.e. we were not looking for differences between “extreme” groups (e.g. lecturers vs. freshmen) and c) the confirmation of the results by statistical means.

The first hypothesis is based on several findings in the expertise research literature: novices use their conceptual, declarative knowledge in the first place for solving problems. Experts also have problem schemes at their disposal. For novices the amount of conceptual knowledge should be a predictor for their problem solving performance. For experts it is rather the amount of problem schemes.

In respect to the ‘quality of knowledge’ there is no definite hypothesis. The two central questions were: 1. Do different qualities of knowledge have stronger or weaker effects on the problem solving achievement? and 2. How closely related are types and qualities of knowledge?

### **Design of the study**

The reference-group of our analysis is the intensive course physics of the German “Gymnasium”. The level of competence corresponds to an intensive beginner college course in physics. Altogether 138 persons were asked to take part at the study. The participants did not get course credits at school but their time exposure was compensated with a small amount of money. Each participant voluntarily spent six hours. The subject of the study was particle dynamics with special consideration to celestial mechanics.

All tests were paper-and-pencil tests and each participant had to work alone after he or she got general instruction from the investigator and written instructions in addition concerning the different tests. The main measuring instruments were a problem solving test for determining the degree of expertise (problem solving achievement) of the participants, a conceptual knowledge test and a problem scheme test (sorting task) for determining types of knowledge and a concept mapping test for determining different qualities of knowledge (see table).

**Table: Measuring Instruments, variables and abbreviations used in the following**

([t]: types of knowledge, [q] qualities of knowledge)

measuring instruments	knowledge variables	abbreviations
problem solving test	problem solving achievement	PSA
conceptual knowledge test	conceptual declarative knowledge [t]	<b>CK</b>
concept mapping test	interconnectedness [q]	<b>N</b>
	hierarchical structure [q]	<b>H</b>
	completeness [q]	<b>C</b>
	degrees of abstraction [q]	<b>A<sub>1</sub> and A<sub>2</sub></b>
problem scheme test	problem scheme knowledge [t]	<b>SK</b>

The participants had to solve 16 problems in the problem solving test. The problems were of varying degree of difficulty and had to be solved within about two hours and without any aids except a simple calculator. (Example: *A recently discovered planet of mass  $M$  is provided with three moons. The moons have equal masses  $m$  and move on a common circular trajectory of radius  $R$  around the planet. Additionally, it is observed that the moons form an equilateral triangle at each moment. Calculate the velocities of the moons. Data:  $M = 10^{24}$  kg,  $m = 1,7 \cdot 10^{20}$  kg,  $R = 67000$  km*) Questions concerning facts, definitions or typical examples had to be answered in the conceptual knowledge tests (examples: a) *Name the Kepler laws.* b) *Describe precisely two different situations in mechanics where closed paths occur.*) In the concept mapping test central concepts of the topics were given and the participants had to connect these concepts with each other and also had to name the relations between each pair of connected concepts. It is possible to determine different qualities of knowledge by a) analysing structural features of the maps (e.g. number of knots, number of links), b) analysing the kind of relations between concepts (correct/wrong, qualitative/quantitative etc.) and c) comparing the maps with reference maps aiming at a description of the hierarchical organisation or the completeness of a map. In a sorting task participants had to sort problems according to solution similarities without solving them. In the most famous work in expertise research Chi et al., (1982) found with this method qualitative differences between experts and novices. They compared lectures, PhD-students and freshmen and found that novices sort problems according to surface features of the problems, e.g. they sort all problems with inclined planes together. In contrast, experts are sorting according to physical principles and solution-relevant situational features of the problems. As Chi, it is assumed in this study that the sorting is done by a comparison with a present cognitive scheme, a so called problem scheme (VanLehn, 1989), in which the characteristics of a problem type are memorised. If there are no appropriate schemes available, the participants sorts the problems according to other criteria which are irrelevant for the solution. All measuring instruments and instructions have been tested and improved before the study. The analysis of the tests e.g. judgment of statement or solution protocols were conducted by persons experienced in the domain of physics under utilisation of detailed directions of analysis. In these proceedings it is not possible to go into more details concerning the construction and analysis of these tests. Much more information can be found in Friege & Lind (2006).

## Results and interpretation:

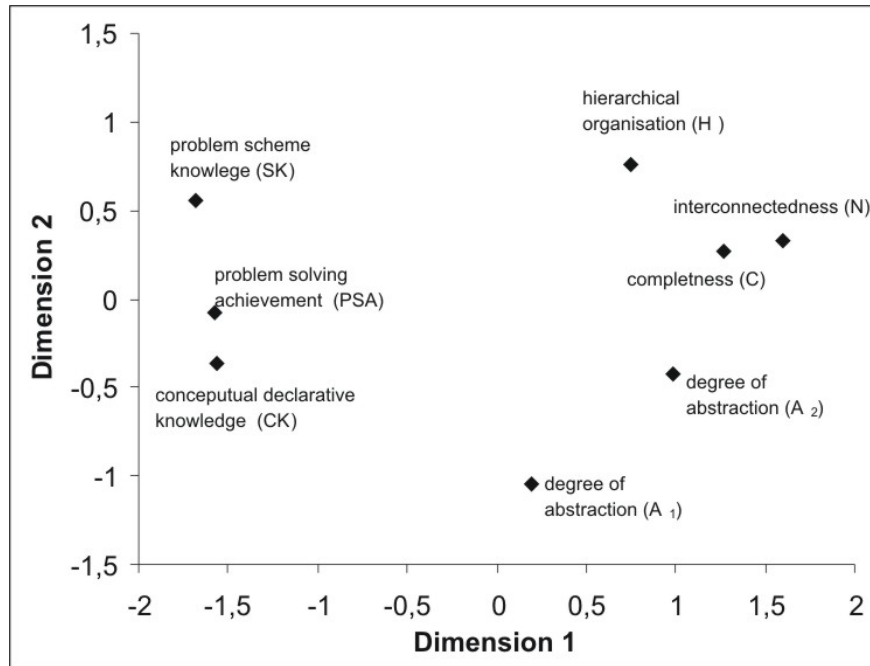
In order to make correlation and regression calculations, it had to be examined, if there are linear relations between the variables under investigation. Indeed, a linear model for all relations turned out to be very good. Since the sample in the study had been chosen in a way, that the variance of knowledge variables and problem solving achievement is very high, it is unlikely that non-linear effects for reasons of restricted variance are overlooked. In the following, the data analyses are therefore based on the linear model.

**Result 1:** It is found that conceptual declarative knowledge and problem scheme knowledge are excellent predictors of the problem-solving achievement and explain more than 80% of the variance of problem-solving. Both predictors practically get the same weight in the regression analysis. Although each of the two predictors explains a big part of the variance of problem-solving already, the explained variance increases again while adding the second predictor. In contrast, adding further predictors taken from the group of the quality variables effects no significant gains.

If only the five quality variables are introduced into a regression analysis as predictors for the problem solving achievement, it is proven that the hierarchical organisation (H) and the level of abstraction ( $A_1$ ) turn out to be the best predictors of PSA. Proceeding step-by-step, however, an additional variable is involved (the completeness C), which has a negative  $\beta$ -value. The remaining two variables are proven to be redundant. Good problem solvers construct concept maps which are of high hierarchical organisation, but show a relatively low level of completeness. Obviously they have a good knowledge regarding the importance of relations in physics and can distinguish 'important' knowledge from 'marginal' knowledge.

A clear picture of the relationships between the different tests is achieved by a multidimensional scaling analysis (MDS). The MDS defines the subjective resemblance or dissimilarity between 'objects'. In this case, objects are tests which are thought to measure certain aspects of knowledge and problem solving ability.

**Result 2:** A 2-dimensional model was chosen. The solution is shown in the figure below. In the case of dimension 1 an interpretation as "problem-solving relevance vs. structure of discipline" seems to be likely. The dimension separates the variables in two groups, which are far apart from each other. The first one is described by the two types of knowledge which are relevant for problem-solving and by the problem-solving achievement. (The distances between SK and PSA and between CK and PSA are small, i.e. types of knowledge are highly relevant for problem solving – as expected by result 1.) The second one is described by the different measures of quality which concern the conceptual structure of knowledge. Problem-solving constitutes the origin of dimension 2. The value of a variable on this dimension seems to be not directly related to problem-solving. A possible interpretation is the following: "single knowledge vs. knowledge which is organised in more comprehensive units". The conceptual knowledge CK and the degree of abstraction  $A_1$  and  $A_2$ , which are related to CK, are assigned to single knowledge. Indicators of the organisation of knowledge are the problem scheme knowledge SK and the measures of interconnectedness N, hierarchical organisation H and completeness C.



**Figure:** 2-dimensional MDS-solution for the whole sample

Calculating an MDS for our whole sample, it is assumed that it is possible to find a solution which applies for all partial samples. This assumption is justified by the good adaptation of the linear model. According to the theoretical background of this study, there should be, however, differences between experts and novices.

**Result 3:** The distribution of the whole sample in experts and novices was done in accordance to the median of the problem-solving achievement. Despite the halving of the sample's size the fit of the MDS model is in both cases visibly better than in the whole sample. The image of the experts is qualitatively quite similar to the image of the whole sample. The most significant differences are: a) the problem schemes are apparently more important for experts when solving problems than the factual knowledge is, b) a distinction between the different quality measures, interconnectedness, hierarchical organisation and degree of abstraction in the case of experts is not reasonable. For novices the five quality measures are placed in a quite similar way as in the whole sample. The most significant difference is the way in which problem-solving is situated in relation to the measures of knowledge. Novices use conceptual knowledge for solving problems; in particular this includes the relations whose degree of abstraction is defining  $A_1$ .

## Conclusions

The investigation was aimed at the identification of characteristics of problem solving relevant physical knowledge. On a more general level, the study yields results concerning the quality of the measuring instruments (e.g. how to determine the degree of expertise with economic means, what do the instruments, e.g. concept maps, measure?). The first result concerns the development of expertise. It seems that expertise is a continuous process, the alternative of a step-like development seems not to be true in a domain like physics.

It is found that both types of knowledge, conceptual knowledge and problem solving knowledge, are excellent predictors of the problem-solving achievement and both are similarly important. However, a more detailed analysis shows that in the case of different participants one of the types of knowledge always heads the process of problem-solving. Regarding the qualities of knowledge the relations seem to be more complex. But what do the quality variables measure? The results indicate that they characterise a different kind of knowledge, which is not identical to knowledge of experts defined by the problem-solving achievement. It could be knowledge concerning the structure of the discipline, which enables one to glance over the concepts of a domain and to develop relations between them. Probably, it is oriented by the structure of knowledge as it is written down in physics textbooks. Knowledge relevant for problem solving and knowledge about the structure of the discipline, do not exclude themselves from one another. However, a lot of students were able to describe the structure of the discipline very well without having a problem-solving achievement of the same level. The reverse case was found more rarely. This can be considered as consequence of physics classes where usually more importance is attached to the structure of the discipline than to problem-orientated application of knowledge. If it is the aim to achieve both in classes, both have to be practised. From this point of view it seems to be a defect in education that – at least in physics - lessons are often predominantly orientated to the structure of the discipline. This perspective ought to be supplemented by lessons which are orientated on the canonical applications of the theory.

### **Acknowledgement**

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# THE WEAKEST LINK? SUPPORTING THE POSTGRADUATE TEACHING ASSISTANT

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The themes for SMEC2006 pose two interesting and many-faceted questions: ‘How essential is a good foundation in Mathematics for the teaching and learning of Science?’ and ‘How well do teaching methodologies translate between Science and Mathematics?’ Yet who are the persons or communities to which these questions are addressed?

Within Higher Education, a major part is almost always undertaken by the Postgraduate Teaching Assistant, who all too often is regarded as the ‘weakest link’ in the system. Yet this is not the fault of the postgraduate student; it is indicative of the fact that this crucial role itself often receives very little attention, and the postgraduates themselves, even less support.

During this talk, we analyse the outcomes from a series of discipline-based workshops run by the Higher Education Academy Maths Stats & OR Network to support postgraduate students with teaching responsibilities. For the most part, these students had attended some form of generic institutional training, yet clearly felt the need for additional discipline-based development. In all 88 postgraduate students attended the workshops, and here we will present details of the typical teaching duties they identified they undertake (the results of which may be surprising), and the support that they felt they required. A measure of the success of the workshops is the high demand for further such meetings in the 2006-07 academic year.

Our presentation will conclude with suggestions and advice which will have direct relevance for anyone with an interest in supporting ‘the weakest link’.

## Introduction

In October 2005 the Maths, Stats & OR (MSOR) Network, a Subject Centre of the United Kingdom Higher Education Academy, initiated a series of workshops for postgraduate students who teach Mathematics and Statistics in universities and other Higher Education (HE) institutions. See <sup>[1]</sup> for a discussion of the first three of these workshops and <sup>[2]</sup> for an account of an individual postgraduate’s experiences of teaching.

As the workshops progressed a unifying pattern began to emerge of the training needs for postgraduates and this is developed further within this article.

The initial programme for the workshops consisted of four sessions:

- Planning and Delivering Small Group Teaching;
- Facilitating Problem Solving Classes;
- Encouraging Participation: Motivating and Sustaining Student Interest;
- Assessing Student Work and Providing Feedback.

An interactive Question and Answer session concluded each workshop and provided participants with the opportunity to seek advice and share advice, and discuss their individual concerns. The workshops were all facilitated by experienced teachers of mathematics within HE, and were offered at three locations within England. As a result, a set of materials have emerged from the sessions providing a substantial and developing resource for subsequent participants <sup>[3]</sup>. There was a good spread of experience amongst the participants who attended these events which ensured a lively and productive exchange of views and ideas.

Almost immediately, other departments of mathematics were requesting similar workshops for their own postgraduates, and a further two events were scheduled for

individual institutions in 2006 with others planned for subsequent years. It is intended that the structure and format of this workshop series will continue to develop and evolve, and feedback from participants and other evaluation mechanisms in 2005/2006 suggested a few changes to the structure of the workshops which now consist of a greater number of shorter, more diverse sessions:

- The First Lesson;
- Planning and Delivering Small Group Teaching;
- Facilitating Problem Solving Classes;
- Presenting & Communicating Mathematics;
- Assessing Student Work and Providing Feedback;
- Interactive Session: Sharing Advice.

During the first phase of the workshops, a total of 88 postgraduates attended. Feedback from participants has been extremely positive: narrative comments indicate that many will re-think how they plan, organise, and/or present their teaching sessions. In our view, the message is clear: no matter how much lecturers and colleagues think about courses they are delivering, for as long as a significant amount of teaching is left in the hands of postgraduate students, they must be appropriately supported and trained, which in our experience they welcome enormously, and this must include a discipline specific component. Otherwise this will remain the ‘weakest link’ in the learning and teaching provision.

### **Background**

A survey of postgraduates’ training needs [1] confirmed that most are involved in running seminars or tutorials or problem classes of some kind in Mathematics or Statistics, 40% are involved in marking exams and about half marked coursework. Postgraduates perform such a wide range of duties directly interacting with undergraduate students, that it was not initially clear how best to support them. However, during the course of the workshops a pattern emerged, with the help of the postgraduates themselves, which seems to provide a natural approach to such training. This is described here, but of course is likely to evolve in response to further feedback. In any event, our main objective was to ensure that all participants had the opportunity to receive targeted advice and support.

Normally postgraduates teach in a small group environment, or are directly involved with either demonstrating how to solve problems, or assisting students in their attempts to solve problems. In relation to assessment, they are normally marking formative student assessment and providing feedback. Therefore, while they are not usually responsible for an entire undergraduate course, postgraduates are actually in the ‘front line’ when it comes to supporting student learning. We can perhaps summarise the basic skills needed by the postgraduates as follows. Postgraduates have to:

- **ENTHUSE** the students about mathematics;
- **ENGAGE** the students in productive mathematical work;
- **EXPLAIN** mathematics to students with varied backgrounds;
- **EVALUATE** student work and mark in a fair and consistent manner.



While the workshops were not initially structured in this way, it naturally evolved as a way to focus on specific skills the postgraduates needed in the context of Mathematics and Statistics. These skills are central to teaching mathematics by any means, and are particularly difficult to master especially by those new to teaching.

### **Enthusing Students About Mathematics**

The postgraduates were encouraged to at least give the appearance of enthusiasm, even if they found a topic uninspiring. In fact, with sufficient imagination there is no branch of mathematics that can't be made interesting. Self awareness and self discipline are needed in order to adapt when you feel you are becoming jaded or boring. A teacher might raise enthusiasm by asking questions such as:

- History of the topic, where did it come from and why?
- Where does it lead to, what areas of mathematics/statistics are built on it?
- What sort of applications does it have?
- Can I derive any of the results in it by different methods?
- Can it be generalised?
- How is it related to my own particular area of interest?
- Why do the students need this, and why should they find it interesting?
- What concepts are at the core of the topic, and where else do these occur? How did the original developers of the ideas find their way through them?

Students are more likely to become enthusiastic about a topic if they appreciate the few core points upon which it is based. Emphasise these to the students so that they have anchors for their ideas, and then motivate and explain why these points are so important, highlighting any interesting features along the way. Provide examples and asides that may be of direct relevance/interest to them (which does not necessarily mean in their own subject area).

The students' curiosity can be aroused by posing 'interesting questions' and then by being persistent in obtaining answers. An occasional short amusing story related to the topic in hand can sometimes spark genuine interest. Sometimes, you can ask the students for their help, for example on something they may have covered in another topic. When talking to the students, try to justify sensibly, as well as logically. Discuss their notes with them. Choose problems that are at 'just the right level'; too easy can be uninspiring, too hard demoralising. If all else fails in eliciting interest, then regular reference to the examination usually does the trick!

As a group exercise the postgraduates were asked to think of a tedious piece of mathematics and then treat it in an interesting way. You will be glad to hear of one workshop at which they couldn't think of anything boring in Mathematics as they loved it all! One group suggested the topic of convex sets, after all what could be more boring than the definition of a convex set? But, why do we need convex sets and what makes them so important and actually very interesting? Well, in a convex

set you can always move between any two points by the simplest path imaginable, a straight line. That sounds quite interesting, surely? Vertices representing optimal points can therefore be located by scanning a convex set with straight lines, again, surely interesting? But how many sets do we know are convex? Aren't they just a rather unrepresentative case and therefore not that interesting really? But can't we always break up most sets into separate pieces that are convex, treat those separately and then put them back together again? In other words, what we can do with a convex set you can usually stretch to any set. The point is that while the dry definition of a convex set may be pretty uninspiring by itself, the teacher can make it much more interesting by providing a few minutes of additional background information.

### **Engaging Students in Productive Mathematical Work**

Postgraduates are often employed in a tutorial or problem solving situation and one of their main tasks is to actually get the students involved, to get them working productively and keep them at it. The importance of preparation in incorporating activity into a session was emphasised. The purpose of the session must be made clear to the student, with succinct instructions for any activity. Learn as much as possible about the students and remember that they will not necessarily be as motivated or able as you.

To establish the right classroom atmosphere start the session off in a business-like way. Ensure that the student has all necessary resources for the session. Indicate some sort of schedule, and set clear ground rules about orderly conduct of the class. Once started, keep things going. Be everywhere and make sure everyone is active. Help them with entry methods to problems. If they are struggling, tell them to try anything at all, and that sensible guessing is fine. Keep to schedule and ensure even coverage of material. Encourage students to use their notes, books, and other resources as appropriate and to create their own learning materials. When working through problems on the board, you are simply the scribe, most of the working should be obtained from the students themselves, and not just the more able ones. As an exercise in engaging students, one workshop incorporated a role-playing session with the postgraduates acting out different stereotypes within a small group environment. The objective of this exercise was to get all students engaged in the activity, even though some had particularly recalcitrant roles! This worked very well and gave the postgraduates a number of tips and ideas that could be used within their own teaching.

If a student is struggling and not making progress, don't rush in to rescue them completely, but provide them a little help along the way. For example in solving a differential equation by integrating factor method many students have difficulty even starting the problem. Don't jump straight in and find the integrating factor for them. Instead, give them a few suggestive products to differentiate, get them to think about the purpose of the integrating factor. For the very weak student, tell them to find an example in their notes and repeat it line for line with the new equation. The overall purpose is to get the student working and thinking, not necessarily to show them how to solve the equation.

Related to the job of keeping the students engaged is the difficult task of maintaining a disciplined and productive working environment. This was raised as an important issue for postgraduates and is sometimes a problem even for experienced staff. As they have recently graduated from students themselves, postgraduates may lack confidence in their role and status. On the other hand they have the advantage that they are 'closer' to the students and may appreciate their difficulties more readily. To assist in keeping things under control, the postgraduates were encouraged to be clear about their duties, responsibilities and status. Set ground rules early on and stick to them. Keep order. Never be rude, sarcastic or derogatory. Allow leave only for essential purposes. Keep on task, and be aware of any diversity issues, for example non-native speakers.

### *Explaining Mathematics to Students with Varied Backgrounds*

Explaining is one of the key arts of teaching [4]. You don't explain by telling, but by listening, learning, and dialogue with the student(s). You first have to find out what the students know so you can 'get on their wavelength', and be sure you understand what 'know' means for your students. Ask the typical first year undergraduate if they 'know' the product rule in differentiation and they will invariably say 'yes'. But whether they will readily recognise its reversal in the integrating factor method mentioned earlier is far less certain. Having established a common language and starting point lead the student through gradually, dangling the next step just close enough to encourage them to move forward, you may have to break off and tell them to go and think about it for a while. Don't be afraid to let them follow a route that may go nowhere; teach them how to backtrack and start again. And when they finally achieve the correct solution, be pleased with them and hurry them on to the next topic. Never use any sort of negative, derogatory or demeaning response to a student's question. Be polite and helpful, but remember, you don't necessarily have to give the student everything they ask for.

The postgraduates were urged to prepare well for their sessions, so that they themselves can solve all the problems in a way in which the undergraduate is expected to solve them. With their superior knowledge they may know a number of different methods for tackling a given problem, but how is the student supposed to solve it? For example an expert may do a Partial Fractions decomposition very quickly by the cover up rule, but have the students been introduced to this? On the other hand, if the student is mastering the subject, then they might be exposed to the other methods to stretch them a little. Postgraduates were therefore advised to develop a 'global' overview of a topic, with a range of different approaches, so they can adapt to different teaching situations as required.

As a group exercise at the workshops the postgraduates were asked to choose a particularly difficult piece of mathematics or statistics, at any level, and prepare a short presentation (5-10 minutes) explaining this topic to a typical first year student in terms they are likely to appreciate and understand. At one workshop a postgraduate proposed a presentation on homology groups, and we began to get worried that the exercise might not work as intended. However, the postgraduate student made an excellent job of it, demonstrating perfectly how even the most difficult ideas can be explained simply by: paraphrasing technical terminology and notation; the careful use of visual imagery; appropriate analogues; and, providing a range of viewpoints, both

converging and diverging. At a more mundane, but no less challenging level, another workshop looked at explaining completing the square to Engineering undergraduates. In this instance, the explanation is usually made difficult because students are not given a clear view of the key ideas involved (expansion of  $(a + b)^2$  and  $A - A = 0$ ) and lack sufficient skills and fluency in the use of these. Therefore, another key prerequisite of good explanation is ensuring that the basic key components are fully understood by the student beforehand.

### Evaluating Student Work and Marking Fairly

The postgraduate participants at the workshops also had the opportunity to mark authentic student work. They marked three students' attempts at expanding  $(2 + 3x)^6$  by the binomial expansion theorem and were asked to discuss what a typical marking scheme might look like for this problem. The table below shows the spread of marks awarded at one workshop (out of 6) for the three examples of typical undergraduate attempts. In the actual exam all students made different types of errors but obtained the same overall mark of 4/6.

Student/Mark	0	1	2	3	4	5	6
A	0	0	2	12	8	0	0
B	0	0	1	3	5	7	3
C	0	1	5	2	6	1	0

The important point emphasised here was that it is necessary to be very clear about the skills being assessed and the learning outcomes being examined. For example, one undergraduate attempt only made the common mistake of omitting the '3' in the expansion, and so all but one of their coefficients were incorrect. However, this cannot be penalised too strongly because they had remembered the binomial theorem formula, evaluated the binomial coefficients correctly, obtained the correct powers, and yet appeared to get 'everything' wrong because of one slip at the beginning. Some postgraduates thought lack of explanation should be penalised heavily, but there was no hint in the question that this was called for. Discussing such issues the postgraduates came to see the importance of a good marking scheme. If nothing else this exercise taught them that there was plenty of room for debate about marking even the simplest of questions, and indeed in their feedback a number of them said they would mark more thoughtfully in future.

### Some Lessons Learned

Formal question and answer sessions were introduced in the second and third workshops, with postgraduates submitting beforehand their most pressing questions so that all participants left the session feeling that their individual concerns had been addressed. This provided a lot of material for discussion. The questions and suggested responses (on issues of small group teaching and running problem classes) for the second workshop were collated, written up and distributed to all delegates and can be

found on the Network's web-site <sup>[3]</sup>. They are most likely questions that every postgraduate teaching assistant will have at some time or other.

In their feedback the delegates gave clear directions for the sort of training they value:

- Lots of practical and hands on exercises;
- Opportunity for sharing ideas and engaging in discussion with others;
- “Nuts and Bolts” no-nonsense explanation and advice;
- Obtaining answers to their individual questions;
- Unified overview of teaching and learning issues;
- Emphasis on the importance of teaching;
- Enthusiastic and dynamic delivery.

We believe we have made a good start in supporting this range of provision and there is little doubt that there is a substantial need to be met as more and more institutions are requesting these workshops for their postgraduate students. Also, there were some suggestions for other sessions the postgraduate teaching assistant would like to see, for example sessions devoted to a wider range of topics such as the special considerations involved when using IT to teach mathematics or sessions designed around the needs of non-native speakers.

### **Postscript**

The workshops described in this article were funded and delivered by the Mathematics, Statistics and Operational Research Network, a Subject Centre of the UK Higher Education Academy. As such they represent a small, but important part of an overall aim of promoting, disseminating and developing good practice in learning and teaching across UK Higher Education. Further details of the work of the Network may be found on our website <sup>[5]</sup>. The Network is keen to hear of related initiatives, whether in the UK or overseas, and publishes a wide variety of contributions in a regular Newsletter ‘*MSOR Connections*’ <sup>[6]</sup>.

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# MOVING CLOSER TO SOLVING THE ‘MATHEMATICS PROBLEM’ WHERE IT ARISES: THE INFLUENCE OF THE ‘TYPICAL’ IRISH PRE-TERTIARY MATHEMATICS EXPERIENCE ON UNDER-PREPAREDNESS AMONG NUMERATE ENTRANTS

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The contribution of mathematics proficiency to the numerate (e.g. science, engineering) disciplines is well-documented. Internationally concern exists that graduates of these disciplines are persistently demonstrating mathematical deficiencies i.e. the ‘Mathematics problem’. A connection has been made between this problem and the worldwide reportage of numerate entrants to numerate courses demonstrating insufficient and incomplete mathematical knowledge. While the characteristics of ‘at risk’ students are more or less equivalent internationally, the same consensus does not exist in relation to the causes of this phenomenon. There is growing consensus however that ‘under-preparedness’ is caused by the apparent mismatch between the nature of the pre-tertiary and subsequent tertiary level mathematics experience. This study focuses the ‘Mathematics problem’ from an Irish perspective, exploring the nature of the ‘typical’ pre-tertiary mathematics experience, which in turn provides insight into the nature of the transition required of Irish students entering the numerate disciplines. Brousseau’s ‘didactical contract’ is used as an intellectual tool to uncover the features of the mathematics experience in two case classrooms in Irish upper secondary schools (Senior Cycle). While the authors are both professional mathematics educators and therefore acutely aware of prevalent classroom practice, the restrictive nature of contract and its implications for students’ future mathematics education left all concerned astounded.

## 1. The ‘Mathematics Problem’- a global issue!

Mathematics has always been noted for its potential to prepare students for life, work and further education. Undoubtedly we all need mathematics at some stage of our lives. In more recent years, however, mathematics education has been deemed a priority among education systems worldwide for its potential to foster advanced thinking skills strategies and qualities (e.g. perseverance, problem solving) essential within the numerate disciplines (e.g. science, engineering) [1]. In light of the economic implications, it is understandable that the existence of substandard mathematics ability among numerate graduates i.e. ‘The Mathematics Problem’ is a source of international concern (e.g. U.K., U.S.) [2, 3]. Consensus exists that this phenomenon is directly related to the fact that numerate entrants, in many cases, are deemed ‘at-risk’ or ‘under-prepared’ i.e. demonstrate mathematical skills deemed inadequate for courses [2, 5]. Ireland is no exception to the rule. While there is a dearth in the volume of research in this area, a number of Third Level institutions (e.g. Cork Institute of Technology, University of Cork, and University of Limerick (U.L.)) have reported substandard mathematics facility among entrants from the mid-nineteen eighties onwards [2, 4, 5, 6, 7, 8].

Internationally (e.g. U.S., Australia, U.K., Ireland) much of the mathematics that ‘under-prepared’ students find difficult astonishes lecturers. Frequently these students demonstrate substandard numeric/algebraic facility as well as limited problem solving skills. Gaps in mathematical knowledge are also prevalent (e.g. trigonometry and

complex numbers) [2, 5, 6, 8, 9]. It has been proposed that many ‘at risk’ students do not possess the mathematical skills required to cope with everyday life not to mention studying Mathematics in college [2, 6].

What causes further unease is the fact the terminal school examination results (U.K., Ireland) fail to provide any indication of preparedness [2, 10]. In the Irish context, for example, 31% of U.L. students who achieved Ordinary Level ‘A1’ and ‘A2’ grades at Leaving Certificate, were diagnosed as being ‘at-risk’ [2]. Universities also report that grade depreciation is a reality [2, 7].

Third level institutions have little alternative but to ‘pick up the pieces’ i.e. such students must reach the same standards of excellence despite their ‘substandard’ preparation in Mathematics [3]. In order to maintain exit standards upon course completion, a growing number of institutions internationally have reacted positively by placing support strategies in place [11]. Approaches vary from adaptation/lengthening of courses to testing and intervention initiatives [10, 12].

The authors’ work within the U.L. initiative (testing and intervention) provided invaluable and unexpected insight into the nature of the problem [13]. Prior to this, the authors, like so many others, would have associated failure in Mathematics at Tertiary Level exclusively with a lack of commitment on the part of students. This insight into the causes/sources of mathematical under-preparedness challenged the authors’ preconceptions and subsequently prompted the initiation of an investigation into the nature of the problem within the Irish context [14].

## **2. The Uniqueness of the Irish ‘Mathematics Problem’**

While a thorough review of the relevant literature highlighted that internal factors within Tertiary Level institutions can potentially trigger or exacerbate poor mathematical performance (e.g. large class groups, lecture format), the vast volume of evidence suggests that the inability of many students to make a successful transition to tertiary mathematics is the primary cause [2, 15]. The fact that Irish Third level entrants are diagnosed as ‘under-prepared’ despite the reality that First Year Mathematics Courses at Tertiary Level generally overlap and extend the school Leaving Certificate Syllabus, supports this belief [2, 4].

When investigating the transition issue in further depth, the uniqueness of the ‘Irish’ Mathematics problem soon became apparent. While the U.K. universities have pinpointed the increasingly heterogeneous nature of the student population i.e. acceptance of vocational entrants and mature students etc. as the primary cause of mathematical under-preparedness, Irish entrants do not exhibit the same degree of diversity. Although the entrance routes to Irish Third Level institutions are now more varied than ever before (e.g. Post Leaving Certificate (P.L.C.)/FETAC awards; Mature Students), the Irish system is still deemed homogeneous in nature i.e. the vast majority of students enter through the State School Leaving Certificate/C.A.O. system [2, 16]. Acknowledging such differing contexts, the authors sought to answer the following question: Why are Irish students exhibiting the same problems on entering university Mathematics courses as their U.K. counterparts?

Given the fact that Irish students seem to have two distinct advantages over their U.K. counterparts; namely the fact that they are obliged to study mathematics for the entire duration of their secondary school experience (five or six years) and achieve at least the minimum qualification; the sameness of the descriptions of the problem in the U.K. and Ireland was cause for even more alarm.



On reflection the authors concluded that the nature of Irish entrants' pre-tertiary mathematics education i.e. in secondary school directly influences his/her ability to make a smooth transition to Tertiary Level mathematics-intensive courses. Consequently, the authors sought to explore the nature of the 'typical' pre-tertiary mathematics experience (Upper Secondary school mathematics) of Irish pupils and in turn investigate the potential contribution of this experience on students' ability to make a successful transition to Tertiary level mathematics. This fact-finding process did not seek a 'culprit' for the underachievement. The purpose of the study was to gain valuable insight into the closed world of the 'typical' mathematics classroom with a view to suggesting alternative, more effective practices where relevant [14].

### **3 The Study**

In order to identify the unknown factors within pre-tertiary mathematics experiences that determine the extent to which one can successfully make the transition to Tertiary Mathematics-intensive courses, it was essential to collect data that provided an in-depth and realistic portrayal of the behaviours and attitudes of all interested parties (i.e. both teachers and students). Therefore a case study approach was adopted. The study focused on two fifth year class groups (i.e. pupils studying the first of a two year Senior Cycle Mathematics syllabus) selected from a co-educational and a single sex (All girls) School. The selection process was based on the voluntary co-operation of the class teachers and principals of each school. Within this report, the former class group is referred to as the 'Mixed group' and the latter the 'Single Sex group'. All concerned were assured of total confidentiality regarding the location and identity (i.e. the use of pseudo-names). T1 and T2 were the codes given to the 'Mixed' and 'Single Sex' teachers respectively.

As the authors wished to explore possible variations in either teacher/student attitudes or behaviours that could possibly be associated with ability, a variety of levels was requested. While the Single Sex group was studying the Higher level course, the mixed group was an Ordinary level class. The teachers were asked to treat the lesson preparation of lessons observed as for any other mathematics lesson they gave.

While it is not proposed that this study is representative of all Senior Cycle mathematics classrooms, due to the narrow focus achieved by a case study approach, the authors believed that it represented a valuable starting point. This study potentially offered all interested parties 'food for thought' and guidelines for future progress and development [14].

#### **3.1 Theoretical framework**

Brousseau's concept of 'didactical contract' was identified as an appropriate intellectual tool to illustrate fully the findings of this study. Brousseau's research suggests that a child's acquisition in a mathematical learning situation is not simply regulated by her level of intelligence, but is equally affected by many other relationships i.e. an unspoken didactical contract exists between the participants of every mathematics classroom determining the roles, behaviours and attitudes of all 'actors' (teacher and students) within the classroom situation [17]. Lim's (2000) interpretation and use of Brousseau's 'didactical contract' were especially relevant to this study. Lim concluded that seven common elements were apparent almost exclusively in the four classrooms observed in that study. The authors could strongly identify with the conditions (i.e. traditional, predictable approach) described in Lim's

‘contract’ and felt that they were indicative of the Irish exam-orientated Senior Cycle mathematics classroom. Such compatibility facilitated the adoption of the ‘didactical contract’ as an appropriate conceptual framework to guide both data collection and analysis of classroom practices and behaviours [18].

## **3.2 Methodology**

**3.2.1. Data Collection.** Qualitative non-participant observation was deemed the optimum strategy for data collection because Brousseau [17, p. 226] reported ‘the hypothetical process of finding a contract is the contract’ i.e. it was essential to undertake an in-depth study of the routine happenings (i.e. behaviours, interactions, and attitudes) of the mathematics classroom. This method allowed the author (MH) to capture all participants in their natural settings in everyday conditions [14]. In order to facilitate participants in overcoming initial caution, it was decided that observation should take place over an extended period (10 weeks). The author was positioned at the rear of the classroom throughout the classroom observations, making no attempt to participate or interfere in activities or classroom discourse. It was hoped, therefore that data collected would reflect participants ‘everyday’ practices i.e. increase validity [19]. Maximum objectivity and validity of findings was also ensured through the use of a structured data collection strategy i.e. observer checklist consisting of factors considered critical in gaining a holistic picture of the classroom environment [14, 19].

A multi-method approach was adopted, to facilitate triangulation and increase the reliability and validity of the findings. Complementary methods included reflection and interviewing. The author kept a reflective journal throughout the observation period. The reflection process, guided by a purpose-developed ‘reflection guide’, took place after every session and facilitated the author with an opportunity to look beyond the description, thus discovering overall patterns and conclusions. Semi-formal interviews were subsequently utilized (after the final observation). This method proved to be an effective means of information backup, providing confirmation and clarification of behaviours observed. Questions also focused on participants’ thoughts, beliefs and attitudes; factors which were almost impossible to detect through observation alone [19].

**3.2.2. Data Analysis.** The data collection process led to the accumulation of a plethora of random information which needed to be analysed. As it was not possible or necessary to examine every action or utterance, the focus was narrowed to factors deemed influential to the nature of the ‘didactical contract’ present in each classroom [19]. Constant comparative analysis ensured that all findings were both grounded and relevant. Initially the establishment of broad codes e.g. ‘Exam reference’, ‘Routine’ pulled the wealth of data into an elementary structure. Through a succession of examinations, the author found that many of the codes were subsets of others and therefore could be merged. Such overlap highlights the richness of the data, as substantial relationships existed between units. The final themes were: ‘Exam-oriented mathematics’, ‘Daily mathematics class’ and ‘Quality of interaction’.

## **4. Findings**

The experiences of two relatively ‘typical’ Senior Cycle Mathematics classes highlighted various practices, which may contribute to the inadequate transition made

by a sizeable number of pupils to Tertiary Level mathematics-intensive courses in Ireland [14].

#### **4.1 Exam-oriented mathematics (Exam Focus)**

Within the Senior Cycle classes observed, exam-focus, whether mentioned or implied by action, was the central concern. Both teachers and pupils alike demonstrated the implicit conviction that the Leaving Certificate Examination is the principal reason for studying Mathematics at Senior Cycle. Consequently the Leaving Certificate exam was regularly used as a motivational ploy to gain and maintain interest and concentration levels among pupils. Comments like *'This is a full question in the Leaving Certificate and it's very easy to do well in if you practice'* (Observation (O) 1, T1) were especially popular and a central component of each lesson observed. Further evidence lies in the fact that the only voluntary pupil interaction within the majority of the lessons observed were queries relating to exam questions.

Pacing of lessons observed was also influenced by the obligation to cover topics required for the Leaving Certificate exam. While a challenging pace was evident in both settings, it was more pronounced in the Honours class. In turn the evident obsession with topic progression affected pupil behaviour. One student linked her passivity with pacing: *'I feel I can't ask the questions I want or need to even though she says 'Well any problems with the homework' ... I just feel she's always giving the impression that she's under pressure to get the course done. It's always as if we have to move on...I don't want to hold the class back'* (Interview (I), T2: Anna).

#### **4.2 Daily Mathematics Class**

The exam-oriented environments observed also determined many aspects of the day-to-day mathematics lesson e.g. the predominant methodology. The statement *'...the same more or less everyday...I'm afraid-quite boring'* (I: T1) depicts the fact that set routine is a central characteristic of both settings. The predominant resources were the blackboard and the primary text. The focus was on the mastery of algorithmic procedures as isolated skills, with only rare connections made to other relevant subjects or every-day links.

Methodologies were traditional in nature and teacher-centred, ranging from exposition to consolidation and practice. Throughout the investigation period, many examples of quick-fix approaches and drill came to the fore. Both teachers adopted a 'reductionist orientation', where teacher was equated with 'effective teller' [20]. Pupils were constantly provided with 'ready-made' mechanisms. One example became evident in a lesson focusing on quadratic inequalities. The teacher stated *'In order to remember the n-shaped graph-remember n stands for negative'* ( $x^2$  coefficient) (O3-T1). Consequently mathematics for these pupils entailed manipulating numbers and letters and filling in the right formula. Students expected a 'learn-off' approach: *'I like the way she goes through the steps and breaks the examples down...'* (I, T2: Olivia).

'Problem-solving' in these settings was limited to practised text-based story problems with one 'right' answer. The limited nature of the students' problem solving skills became apparent from the typical pupil's response when problems strayed slightly from the textbook format: *'It's grand doing all the section questions, but when they start mixing topics, it's impossible to know where to start'* (I, T2: Marian). Confirmation of pupils' 'situated learning' came from one of the teachers, who stated *'If they couldn't do an exercise that is slightly different, they wouldn't try- the majority would leave it blank...'* (I: T1) [21].

Another source of concern was the fact that none of the pupils interviewed had ever completed practical or investigation work in their post primary mathematics education to date.

### 4.3 Quality of Interactions

Dialogue, for the most part, was teacher-initiated. As questions generally lacked direction e.g. ‘*Anyone not get it?*’ (O1-T1), the opportunity to assess the level or even presence of understanding was lost. The statement ‘*Sometimes you feel like you’re taking it down like a robot. You’re really not involved in it...*’ (I, T2: Olivia) reflects pupils’ beliefs that their role was primarily a passive one of listener and copier. Students were largely unwilling to publicly share their thoughts for fear of making an error and facing public embarrassment. Two-way and even pupil-initiated interaction was only plausible during the ‘practice and consolidation’ stage of the lesson.

In their effort to promote positive attitudes, both teachers repeatedly cajoled the groups, offering endless positive reinforcement regardless of relevance or accuracy. In her bid to encourage the pupils, the Ordinary Level teacher demonstrated very flexible expectations. As a result, an inability to recall even the most basic elements of previous topics was deemed acceptable: ‘*...They expect to be spoon-fed at all times, not attempting homework if it looks too hard...*’ (I: T2) [14].

### 5. The nature of the ‘didactical contract’ in the classrooms investigated

The collection and analysis stages facilitated the formulation of a ‘didactical contract’, which was representative of the actions and attitudes of all participants in both educational settings. This unspoken contract reflected a negotiated agreement between the participants observed, consisting of both consensual and involuntary demands on all concerned. While much of the contract conditions represented both classrooms, some variations between the two groupings became apparent. The author utilised the word ‘should’ throughout the contract, signifying that these deductions are objective interpretations of the data analysis process. The common elements of this ‘didactical contract’ are as follows:

- The Leaving Certificate terminal exam should be the central aim of the Mathematics class. This exam should be the core component of each lesson, present as a sole motivation to learn a new topic. The teacher should present work referring to its inclusion/importance in the Leaving Certificate and provide details of the gain/ loss of marks at every opportunity. All class tests should be based on previous exam questions. No time should be wasted in class, as the adequate chapters for exam preparation require completion in the shortest possible time.
- The teacher should introduce the lesson by correcting the homework swiftly and orally if possible and move onto a new topic. Blackboard work should be curtailed, unless pupils have major problems and ask for help. During the homework correction, the teacher should interact mainly with the group, asking for the answer to the exercises. The introduction of a new topic should consist of the illustration of a number of worked examples on the blackboard. After pupils have copied these into their copies, they should practise similar exercises from the course text entitled, *Text and Tests*. During individualised practice work, the teacher should circulate, offering personal feedback and help to all needy pupils. The pupils should use this opportunity to ask questions regarding the homework etc. Before the bell, similar type exercises should be set for homework from the primary text. Homework should be attempted in order for pupils to gauge how much they actually know.

- The teacher should not depart from the set lesson routine unless the class are preparing for a test, in which case the entire class should be used to review formulas, techniques and standard problems. The teacher should, if possible, get something new done everyday, as pace is vital. The standard lesson should never include practical work or investigation or make logical links to other subjects or everyday life unless it is directly relevant to the exam e.g. Statistics.
- The teacher should not ask pupils complicated individualised questions, as the pupils should not be publicly scrutinised. Such questioning in class should be pitched at the group, to allow more outspoken and well-able pupils to volunteer. The only directed questions that should be permitted are those on new topics or on straightforward workings.
- The teacher should ‘break things down’ and simplify for the pupils. The pupils in turn should listen and try to learn off. The teacher should provide pupils with tricks to remember mathematics methods and a step-by-step breakdown of problem-solving techniques i.e. the use of trigger phrases. Pupils should not be expected to persevere with difficult questions or to be able to solve questions requiring a combination of procedures.
- Pupils should not be expected to remember previous topics and the teacher should be patient and ready to re-explain any section/method on request. The pupils should listen and learn, participation in class activities is not vital. Once the minimum standard is achieved i.e. homework attempted, on task in class and able to answer directed questions, the teacher shouldn’t request any more. The teacher should always be positive and encouraging even if pupils are not working to their ability.
- Any individual teacher or pupil should not interfere with the contract described above, even if they are unhappy with many elements of it. Pupils should not interrupt the lesson unnecessarily, thus holding back the group, even if they are confused. Pupils should work on passively in class even if they require extra blackboard reference, examples or time on a particular topic. The teacher should not ask directed questions that may demand thought and reflection, or set questions which have more complex wording or layout to that of the main text. The class form should not be disrupted, as the present momentum is deemed vital in order to keep on target in the pursuit of exam success. All loose ends and confusion should be dealt with during seatwork or after class [14, p. 159-161].

## **6. Reliability of the Findings**

The authors have already acknowledged that this study does not provide a comprehensive picture of the pre-tertiary situation and undoubtedly there is ample scope for further national and international research into this phenomenon. Despite this fact the study does provide invaluable insight into the nature of the ‘Mathematics problem’ in Ireland and is consistent with the concern and discontent evident among the relevant research in the field.

Agreement exists within the relevant literature that the Irish Second Level Education System is extremely ‘exam-oriented’ [5, 14, 22]. The powerful backwash effect of the Mathematics Leaving Certificate examination on what and how it is taught is an ongoing cause for concern [22]. The fact that the terminal examination is the sole means of assessment causes many teachers to utilize ‘course dilution’ i.e. omit sections of the syllabus for the purposes of examinations [4, 5]. This approach directly contributes to the existence of gaps among numerate entrants’ knowledge [8].

National and international studies also concur that exam-focused teaching within the Irish context is ‘traditional’ in nature, prioritising recall and routine procedures, while

pupils remain largely passive, relying on rote memory and special-purpose algorithms as alternatives to understanding [2, 4, 8, 23]. The textbook has been found to more influential in planning than the curriculum [22]. The lack of reference to the potential role of the subject to pupils' own lives or other related subject areas has also been deemed unsatisfactory [8, 23]

Data also exists on the effects of the implemented curriculum (i.e. the attained curriculum). From a national perspective, the Chief Examiner's Report (2000) was especially critical of the serious decline in ability, effort and understanding among Leaving Certificate students at all levels. A source of particular concern was many students' apparent inability to demonstrate relational understanding [24]. In more recent years the N.C.C.A. (2006) report that "...many students leave school with only a superficial understanding of the subject and little or no conceptual knowledge" [8, p. 36].

The mathematical performance of Irish Second Level pupils in the various international studies of achievement e.g. Programme for International Student Assessment (PISA) has been consistently 'satisfactory'. While Irish pupils perform better than the international average on tasks involving basic mathematical operations, their more limited ability to use higher-level mathematical thinking e.g. problem solving (required by the numerate disciplines) is a particular cause for concern [25, 26].

## **7. Food for Thought**

While there is no doubt that Mathematics teachers, alongside their peers, are endeavouring to do what's 'best' for their pupils, this study illustrates that the present obsession with 'exam-oriented' practices serves to narrow such pupils' future potential. This and other studies strongly suggest that the prevailing inflexible and unresponsive learning environment promotes mediocre learning and poor study habits; is serving the students and economy poorly [27]. The reality is, however, that there is no incentive to change because students can achieve high standards in the predominantly abstract, context-free Leaving Certificate examinations which focuses predominantly on memorisation, the practice of technique and ability to spot cues [8, 14, 23].

While Mathematics-intensive courses at Tertiary Level need independent learners possessing conceptual and transferable skills required to solve unfamiliar problems, the development of these essential skills is not fostered within the classrooms studied. Unfortunately the pressure on schools to deliver good examination results seriously weakens the greater aim of providing a high quality mathematics education [28].

## **8. In Pursuit of Change: Aspirations**

If Second Level mathematics education is to adequately prepare pupils to participate in mathematics courses at Tertiary Level, they must leave this level of their education in a 'high energy state' i.e. demonstrate a predisposition to ...confront any problem given to them/invented by themselves with their previous learning in Mathematics, in an active and accessible state and with the assumption that they have the ability to progress [29, p. 105].

If our pupils are to demonstrate this ideal mathematical state, considerable changes in the quality of Second level mathematics provision are inevitable.

Firstly, all interested parties (e.g. practitioners, Department of Education and Science) must become fully informed of the importance of mathematics and the current phenomenon, and realise that pupils' achievement in the Leaving Certificate mathematics examination is only one aspect of their mathematics education. Widespread awareness of the prerequisite mathematical skills and abilities required by pupils hoping to enter Tertiary Level mathematics-intensive courses would prove invaluable [8, 14].

Second Level mathematics practitioners must alter the predominant classroom practices. A move away from over-reliance on traditional approaches is a must. It is necessary that the belief that mathematics is useful and vital for all people is portrayed through the use of integration and real-world connections. There should be an emphasis on 'connection-making' i.e. moving from the known to the unknown, thus promoting understanding. Pupils need to experience open-ended relevant problems, which require perseverance and risk-taking. In such an environment mistakes and confusion are considered as opportunities for learning, rather than a source of embarrassment or discomfort that should be avoided if possible [14].

The authors are cognisant, however, that many of the contributing factors associated with the predominant examination-focused practices lie outside teachers' control. A focused alteration in national policy is prerequisite to real change. Consequently, much hope can be gained from the government's recent move to seek a 'root and branch' review of Second Level mathematics education with special reference to Senior Cycle mathematics. The fact that many of the recommendations of this study have been subsequently reflected in the relevant national discussion and consultation papers is also heartening [5, 8].

The National Council for Curriculum and Assessment publications also reflect the authors' belief that the alteration of the mode of assessment, to include project work for example, would foster the development of problem-solving and higher-order thinking skills [5, 8, 14]. There are also proposals to change the curriculum to include 'realistic mathematics' applications and facilitate the development of a range of skills. Support for teachers at all levels has also been deemed a precondition to genuine change. If the authors' recommendations become a reality, the envisaged 'root and branch' reform will move one step closer to tackling the 'mathematics problem' where it arises [8].

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# PRIMARY CHILDREN'S ATTITUDES TOWARDS SCIENCE

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This study revealed that a decline in attitudes towards science and school may begin as early as eight years old. A questionnaire was completed by 647 children between the ages of four and 11 to investigate their attitudes towards school science. The children in this sample were very clear about what they liked and didn't like to learn about in science. In general, girls were slightly more positive about science. Implications for classroom planning and teaching are discussed.

## Background

“The UK will need 2.4 million more people to work in scientific jobs by 2014”  
*[The Confederation of British Industry<sup>1</sup>]*

The year 2014 may seem far away but the ‘people’ referred to by the CBI above comprise the current primary school population. Recent statistics suggest that the decline of science uptake post 16 has already resulted in a drop in the number of suitably skilled young people to work in scientific jobs. If we are to meet the future requirements for a scientifically skilled workforce the focus needs to be placed on the children now in primary schools.

Apart from encouraging children to work towards careers which are ‘economically valuable’, it is very important to focus on the value of science learning which can enable children to develop critical thinking and thus distinguish between fact and fiction (e.g. in advertising, news coverage, computer gaming and magazines).

Findings related to primary school pupils have shown a decline in attitudes towards science among upper primary school children. Murphy and Beggs (2003) reported that 8/9 year olds had more positive attitudes to school science than 10/11 year olds. They suggested that the decline could be related to preparation for national tests, inappropriate curriculum content and lack of experimental work. Pell and Jarvis (2001) reported that there was a ‘year-on-year deterioration’ of attitudes towards science and highlighted how important the primary years of schooling are for the formation of these attitudes. To foster the formation of positive attitudes towards science it is important that we listen to what children think about science in school. We can build on the areas of science that children like most. Reiss (2000) pointed out that science education will be successful if pupils believe it is of ‘personal worth’.

This study considers the attitudes of children in primary school and addresses the following questions:

- What aspects of science are children in primary school positive about?

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<sup>1</sup> BBC News (2006) *Schools 'letting down UK science'* Visited August 2006.  
<http://news.bbc.co.uk/go/pr/fr/-/1/hi/education/4780017.stm>

- Is there a difference in attitudes towards science between younger primary children and older primary children?
- What are the implications of these findings?

### **Method**

The questionnaire was administered to a sample of children in six urban primary schools in Belfast. The sample comprised 48% girls and 52% boys. The questionnaire contained open and closed questions about attitudes towards science and school. All questionnaires were administered by the author during April and May 2006. The questionnaire contained a combination of items relating to:

- enjoyment of science
- importance of science
- attitudes towards some science topics
- attitudes towards school

Items relating to the children's attitudes towards school were included for comparative reasons as children with positive attitudes towards science may just be positive about everything they do in school and vice versa. The questionnaire was adapted for younger children (primary one and two), for instance items which specifically mentioned the word 'science' were omitted or reworded.

### **Results**

#### *Enjoyment and importance of science*

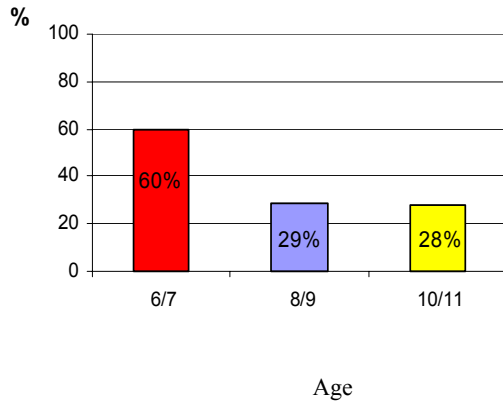
Children's attitudes to school science generally declined with age (Figure 1).

The sharpest decline was in the percentage of children (8/9 years old) who said they liked science lessons more than other lessons [Figure 1(a)]. The percentage of children who said they liked coming to school was less than 60% for eight years olds [Figure 1(c)]. Only 59% of the total sample said they liked coming to school. However, the decline in attitudes towards school was not as marked as the decline in the proportion of children who said they liked science lessons more than other lessons (Figure 2).

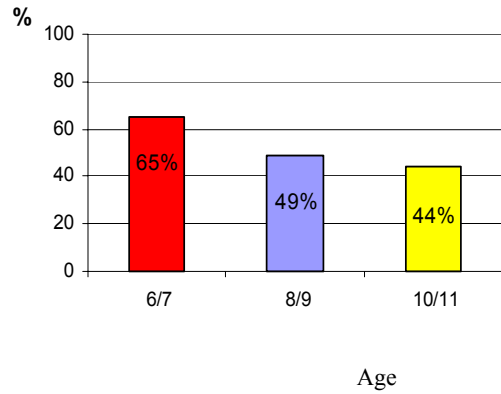
Children were asked if they thought being able to do science, use the computer, read and do sums/number work was important for helping them to get a job when they are older (Figure 3).

Although the importance of reading and sums/number work decreased for children between six and nine years old, it was higher among the older children (10/11 year olds). The same can not be said for computers and science. There was a gradual decline with age in the proportion of children who responded that science and computers were important for getting a job (Figure 3). However, when asked if science helps people (open question), a greater percentage of children aged 10/11 did mention how science can help you to get a job (Figure 4).

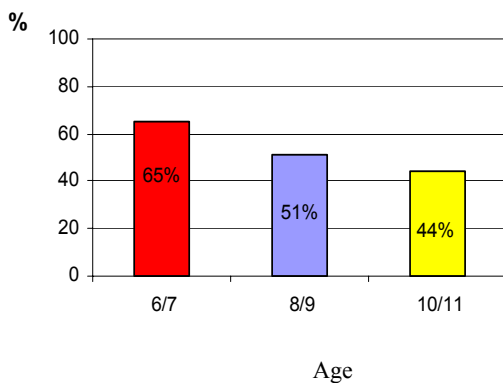
(a) Do you like science lessons more than other lessons?



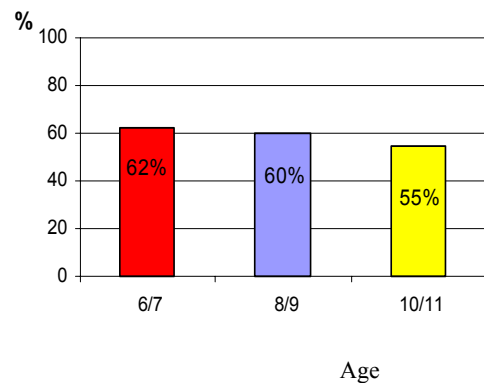
(b) Do you like doing science tests?



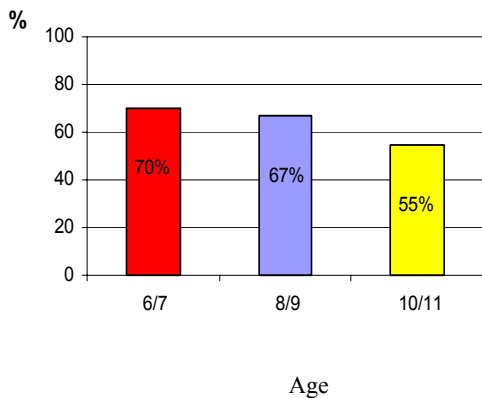
(c) Do you like coming to school?



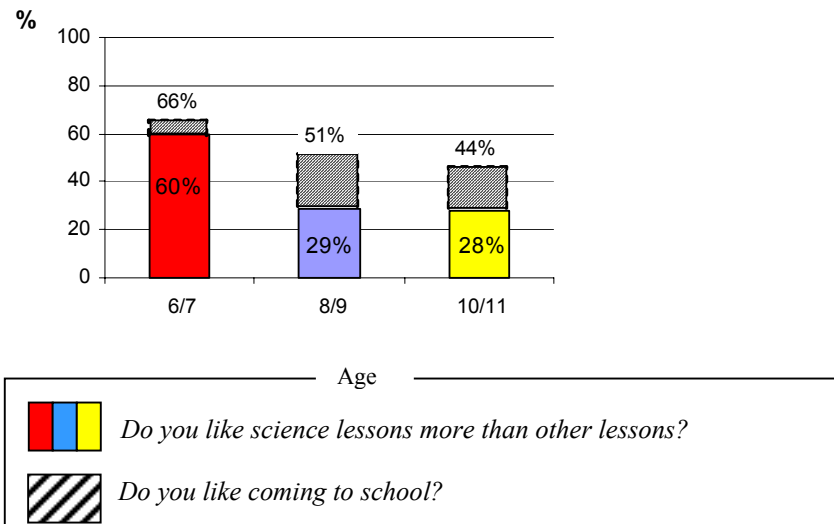
(d) Do you think all children should have to do science at school?



(e) Do you think being able to do science will help you to get a job when you are older?



**Figure 1:** *Enjoyment of science: the percentage of children who replied 'yes'.*



**Figure 2: Enjoyment of science and school: the percentage of children who replied 'yes'.**

For example, these 10/11 year olds children said:

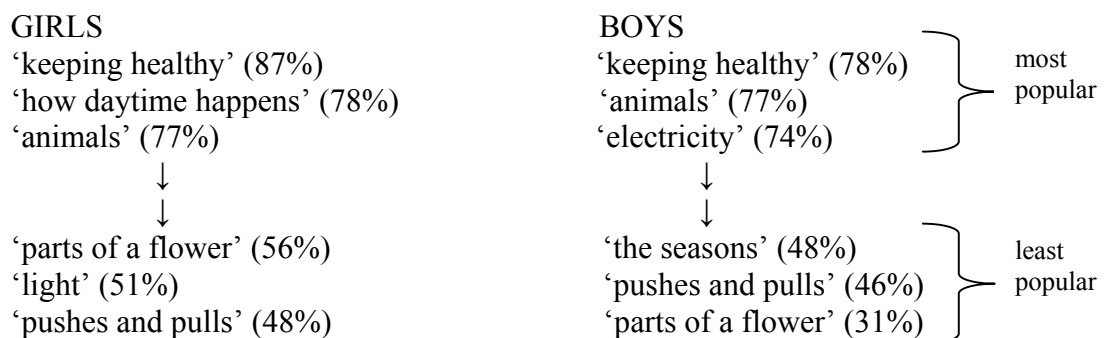
Yes, because it helps people get jobs (boy, aged 10)  
 I think it does help people in a way, because if you wanted to be a teacher you would have to know about science. (girl, aged 10)

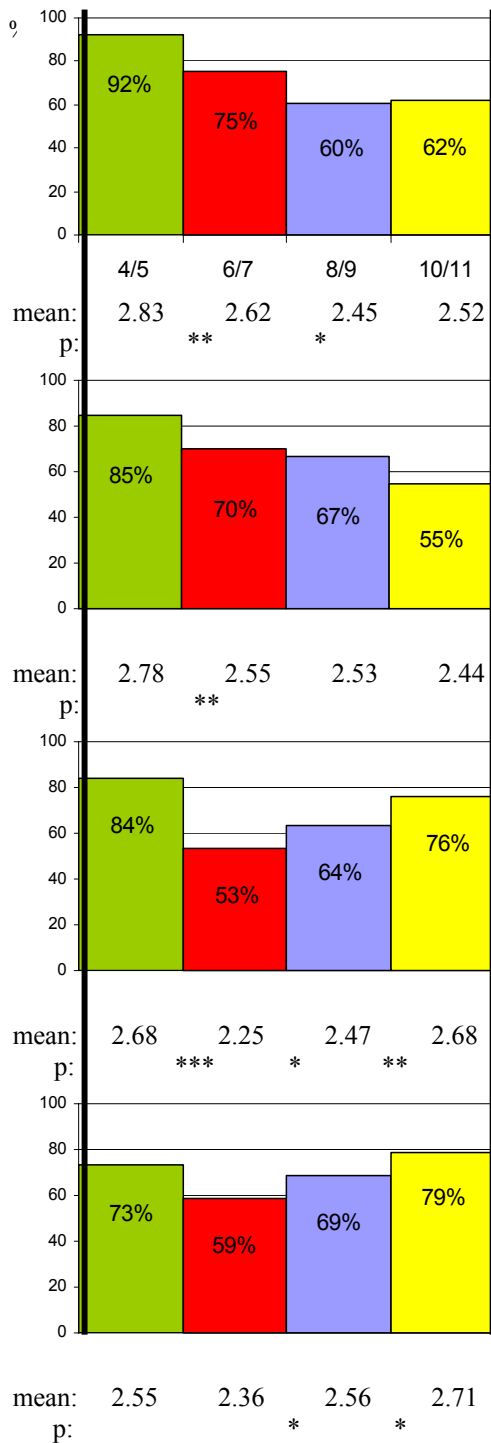
Nevertheless, when compared with 10/11 year olds a greater percentage of younger children (6/7 and 8/9 years old) mentioned how science is helpful academically (Figure 4). For example:

Yes, by learning new things, helps people get smart (girl, aged 7)  
 Yes, because it makes them smarter (boy, aged 7)

#### *Attitudes towards science topics*

The children were asked if they would like to learn more about 18 different science topics. The three most popular and the three least popular reported by girls and boys are outlined below:





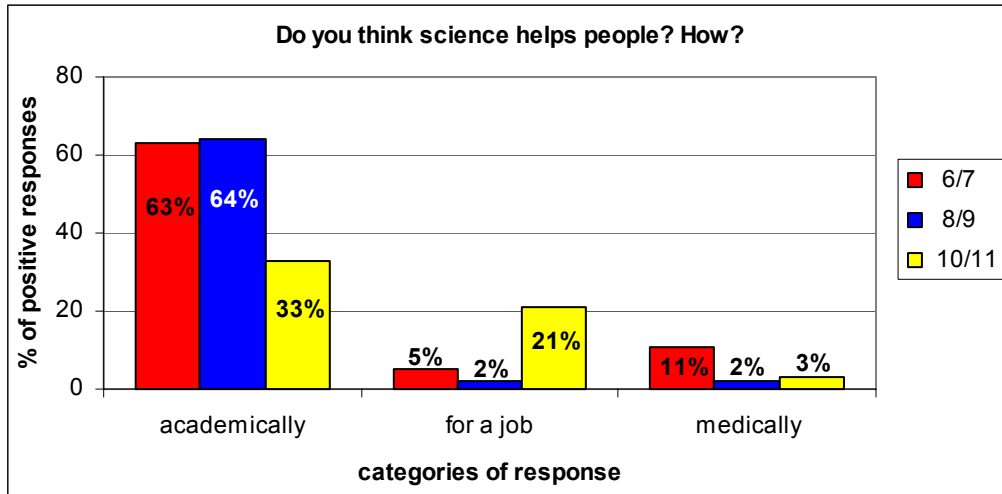
Do you think being able to **use the computer** will help you to get a job when you are older?

Do you think being able to **do science** will help you to get a job when you are older?

Do you think being able to **do sums/number work** will help you to get a job when you are older?

Do you think being able to **read** will help you to get a job when you are older?

**Figure 3: Importance of science (for a job):** the percentage of children who replied 'yes'. The p values represent significant differences between successive age bands [\*sig. at  $p < 0.05$ , \*\*sig. at  $p < 0.01$ , \*\*\*sig. at  $p < 0.001$ ].

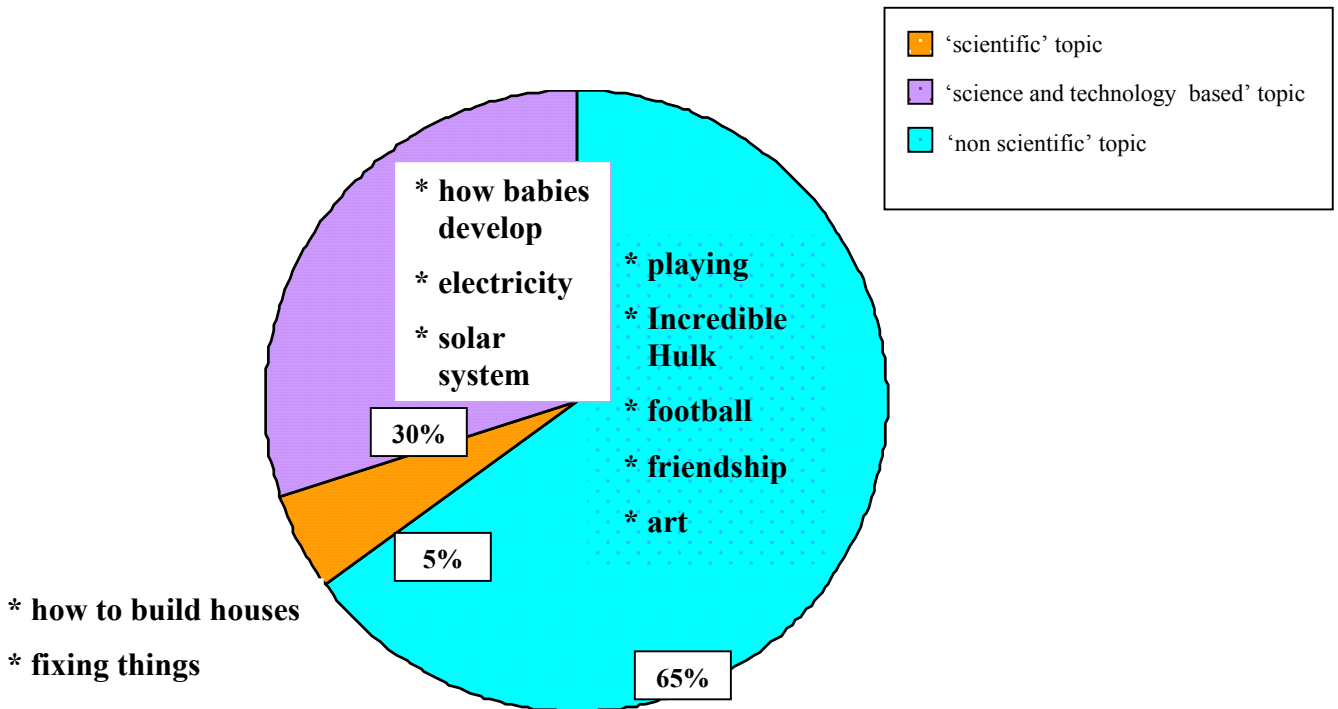


n= 28 children aged 6/7, 57 children aged 8/9, 43 children aged 10/11

**Figure 4:** The percentage of positive responses, by category, given by children in different age groups in relation to how science helps people

Although more girls said they would like to learn more about ‘keeping healthy’, there was very little difference in the order of preference between boys and girls for all 18 topics. ‘Parts of a flower’ was unpopular with both boys and girls yet significantly fewer boys than girls said they would like to learn more about it.

All children were asked to name something that they would like to learn about (not necessarily science) in school. The responses were coded as ‘scientific’, ‘non scientific’ or ‘science and technology based’ topics. The chart in Figure 5 (with examples) shows that less than one third of children named ‘scientific’ topics.



**Figure 5:** What else would you like to learn about in school?

## Discussion

Overall, children in primary school are positive about some aspects of science and less positive about others. Science tests are not popular and this may be why the majority of children do not like science more than other subjects, especially in upper primary school. High stakes testing (the Transfer Test) is said to be one of the factors that has led to a decline in 10/11 year old children's attitudes towards science in Northern Ireland (Murphy and Beggs, 2003). Significantly more 8/9 year olds in this study disliked science tests than 6/7 year olds. A possible explanation for this is that intense preparation for the Transfer Test (including practice *tests*) begins in the penultimate year of primary school when some of the children are still nine years old. Murphy, Ambusaidi and Beggs (2006) suggested that the assessment procedures could account for 'less of a decline in interest and enjoyment in science' among Omani children than children in Northern Ireland. In Oman, there is no national testing for children at the end of primary school. The decline in attitudes towards school was not as sharp as the decline in the proportion of children who said they liked science lessons more than other lessons. This is in keeping with the findings of Morrell and Lederman (1998). They found a weak relationship between attitudes towards school and attitudes towards science and indicated that the decline in attitudes towards science was not 'part of a global attitude problem'. It is therefore important to address children's attitudes towards *science* in particular. Evidence presented in this study therefore reiterates that the primary school years are highly significant with the formation of attitudes beginning as early as 8 (Pell and Jarvis, 2001; Ormerod and Duckworth, 1975).

The decline with age in the importance of science (for a future job) is more marked than for reading and sums/number work. This is a concern given that Blatchford (1992) found that children were aware of future careers before going to secondary school. Although a greater percentage of older children mentioned how science can help you to get a job in their open responses, there was a decline in the responses related to how science can help us to learn and do well in school (academically). Does this mean that primary school children are already dismissing science as part of their future long before (six years) they make their post-16 choices? The focus on increasing the number of young people who choose science careers must therefore concentrate on primary school children.

The importance of computers for future job prospects decreased as much as for science. Given the nature of our technological based society, in particular in relation to science careers, this is also very concerning. Perhaps children do not see computers as valuable for future careers due to their exposure to computer games. Careful planning for the inclusion of ICT and its advantages in science is therefore important. However, more investigation into children's attitudes to computers and ICT would need to be carried out.

Children in primary school are positive about certain aspects of science. In keeping with Murphy and Beggs' (2003) findings, learning about the 'parts of the flower' was among the least popular and 'keeping healthy' was most popular, especially among girls. It is encouraging that the most popular topic (healthy eating) is of current interest in society and has received a lot of focus in schools. Children may feel that healthy eating is of 'personal worth' (Reiss, 2000) and like finding out more about it.



Dislike of learning the ‘parts of a flower’ may be related to the terminology, which is confusing and difficult.

Two of the top three topics were in the physical sciences *how daytime happens* and *electricity*. The negative attitude towards physics in particular does not encompass all elements of the physical sciences. Inspiring boys and girls with the areas they like at an early age may encourage more positive attitudes towards physics as a subject later in their school lives. On the basis of the data from this study it would appear that probing children’s likes and dislikes around science topics may be very useful in planning science lessons. Also it is good practice to probe children’s ideas and understandings at the start of topics (Kerr *et al*, 2006).

The relatively high proportion of children with negative attitudes towards coming to school was unexpected. Similar results were reported by Sa’di (2001) between Jordanian children in grade one (6/7 year olds), grade two (7/8 year olds) and grade three (8/9 year olds). Sa’di (2001) found a significant decline between each grade. However, findings from this paper go further to show that there is no significant decline between 8/9 year olds and 10/11 year olds. Morrell and Lederman (1998) also found no significant difference between groups of older children. However, their children were in middle and upper secondary school. The current study, however, suggests that a significant decline in attitudes to both science and school may begin in primary school.

#### *What are the implications of these findings?*

The attitudes of children in primary school deserve equal attention to those in secondary school. Evidence from this study shows that children are beginning to feel less positive about science (and school) at a younger age than previously thought.

Children as young as four years old are very definite about what they like and don’t like to do at school. It was surprising that during extended questioning lots of four year olds said that they didn’t like playing in the water and painting because they got ‘wet and dirty’. This is an important point to consider when planning how to engage children in certain activities. Young children are capable of providing ‘worthwhile indicators of how they view science’ (Pell and Jarvis, 2001). It is time that we started listening to children in every year of *primary* school. By building upon and nurturing their enthusiasm for the areas of science (and school) that they do like and want to learn more about, we can foster more positive attitudes towards science and its invaluable skills. In particular, young children should be engaged and inspired about the particular aspects of physical science that they like (e.g. electricity) to improve attitudes towards physics as a subject in later school life. This may require planning to include the areas of physical science that are currently not on the primary curriculum (e.g. how daytime happens). In Northern Ireland, greater flexibility in the new curriculum allows for this.

Fostering more positive attitudes towards science at primary level will give the workforce of the future a greater range of choice, to include jobs in science. Education for 'careers' must begin earlier than secondary school.

“If we are to engage and inspire young people with science, it is important that we listen to their hopes and concerns”<sup>2</sup>

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# BRUSHING UP ON MATHEMATICS FOR QUANTUM CHEMISTRY

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Math intensive chemistry courses rely on sound mathematical pre-knowledge. In the first-year Quantum Chemistry course at the University of Amsterdam it was noticed that many students lack basic mathematical knowledge. For instance, they had difficulties giving the derivative of the exponential function.

A remedial mathematics program was set up in the first year to tackle the math problems. During the Quantum Chemistry course students did small on-line mathematics assignments in order to prepare for the lectures. In each of these assignments students trained only in the mathematics needed in the corresponding lecture. Students got enough opportunities to practise, but it was their own responsibility to brush up their mathematical knowledge. In case of a disappointing score, students were able to do more exercises of the same kind. The teacher closely monitored the progress of his students and gave feedback. It was found that the students could follow lectures and tutorial sessions better than in the previous years. This time students were able to follow or do the basic mathematical calculations. However, there were still not enough students who passed for the Quantum Chemistry course. It was concluded that proper mathematical skills are indispensable, but not sufficient for understanding Quantum Chemistry.

## Introduction

Proper mathematical skills are important for every science study. Students entering science studies all over the world are confronted with the difficulties in mathematics due to the transition from the secondary school to the university. The London Mathematical Society published a report in 1995 with several recommendations how to solve this problem<sup>i</sup>.

Lately, the mathematical abilities of the incoming students in the Netherlands dropped substantially. To help their students brush up their mathematics knowledge and skills many universities in the Netherlands offer different additional mathematics courses. A special interest group on mathematics pre-knowledge problems (SIGMA) is being organized in the Netherlands to bring educational institutions and different educational projects closer to jointly work on these problems and to develop new approaches.<sup>ii</sup>

At the University of Amsterdam, an approach is developed to cope with the mathematics problem in the first year of the science studies: Mathematics, Physics, Astronomy, Chemistry and Bio-exact. This initiative is part of two educational development projects with other higher educational institutions in the Netherlands: Web-spijkeren<sup>iii</sup> and MathMatch<sup>iv</sup>. A program is designed based on electronic diagnostic tests and the mastery learning of mathematical abilities by doing many exercises while receiving feedback. To help freshmen to bridge the mathematics gap in the first semester they follow brushing up activities for mathematics. Students take a pre-test in their first week at university to get an idea of their level in mathematics. This test is not an entry exam: students do not get a mark, but an advice to train their mathematical skills based upon their mark. In this training they are supported by tutors. In the fifth week students take a post test, to see their progress.<sup>v</sup>

The results of this test were compared for students from different science studies. Chemistry students scored less well than the rest. For only 50% of them, the brushing-up knowledge activities appeared not to be sufficient to pass the post-test. Their progress however was comparable to that of the other science students.

In the spring semester chemistry students follow a course on quantum chemistry.

Among the topics discussed are: principles of quantum mechanics, structure of atoms and molecules, chemical bonds, and molecular orbital theory (LCAO-MO). The mathematical knowledge and skills are crucial to be able to follow this course. Many students do not have this knowledge and skills ready to be used. Some of them follow the Quantum Chemistry course without passing the fall semester Calculus I exam first. These students confront problems in learning quantum chemistry. To help them, a follow-up to the introductory brushing up mathematical pre-knowledge program was developed for the Quantum Chemistry course.

### ***Math problems in a Quantum Chemistry course***

This initiative resulted from research into the teaching and learning of quantum mechanics for first year science students. In a first inventory round, conducted in 2004-2005, great learning difficulties amongst chemistry students when learning quantum chemistry were found. These students mainly seemed to have difficulties with the mathematics involved. So much in fact that the teacher had difficulties to discuss the main subject of the course: quantum chemistry.

For example, students were not able to give the derivative of  $e^{ax}$ , or even  $e^x$ . Also, students were not able to solve rather simple differential equations, like:

$$\frac{d^2 f(x)}{dx^2} = -af(x).$$

Maybe of more importance, students found it hard to perform calculations with symbols, instead with numbers. This could be seen when derivatives had to be calculated of the form:

$$\frac{d}{dx}(g(y)f(x)).$$

As a result these students had problems following the lecture. Very often the teacher was actually teaching mathematics, instead of quantum chemistry. For students that did not have difficulties with mathematics this was a waste of their time.

Quantum Chemistry is given in the spring semester of the first year. In the fall semester chemistry students follow a calculus course together with students from other science studies. Many chemistry students do not pass the calculus exam. It is not surprising that these students have difficulties coping with the mathematics in math intensive chemistry courses. On the other hand many of the students that did pass the calculus exam also had difficulties following, or doing the mathematical calculations. This indicated that attention should be paid to pre-knowledge of mathematics of students that follow the Quantum Chemistry course. It was also clear that the setup of the Quantum Chemistry course itself needed some attention as well.

### **Tackling the math problems**

It was decided to keep the contents of the Quantum Course the same, and focus on the mathematics pre-knowledge. Some changes were made to stimulate students to prepare for the lectures. For instance, students had to answer a small set of so called warming-up questions on the internet with respect to the content of the upcoming lecture. Shortly before the lecture (“Just-in-Time”), the teacher could collect the students’ answers, and use them in his lecture.

To remedy the math problems, a follow-up to the fall semester brushing up program was organized:

- Just before the start of the Quantum Chemistry course all students had to take an on-line introductory diagnostic test in mathematics. Depending on their score they would get advice on whether or not to participate in the follow-up brushing up program.

Students did the test in their own time, and did not receive any study credits for it.

- For the main part of the brushing up program, a set of on-line assignments was developed for each lecture. These assignments were made in Maple T.A., an on-line computer algebra based testing and assessment environment.<sup>6</sup> Depending on their score on the introductory test, students were advised to take these assignments prior to the lecture, taking about 15 minutes. With enough practice students could get the required knowledge in time for the lecture. However, it was their own responsibility to do the tests satisfactorily.

- The teacher followed their results in Maple T.A. and regularly gave students a comment about it. In case of recurring difficulties with a certain mathematics concepts, an extra explanation was given to the whole group during the tutorial.

- The assignments only consisted of problems dealing with mathematics needed in the following lecture. Most of the problems were directly associated with the lecture slides. For example,

- Figure 1 shows part of a lecture slide with a calculation to normalize a wave function. The steps needed in this calculation are directly translated into a Maple T.A. exercise, see

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- Figure 2.

- Each week a tutor was available for one hour to answer students' questions about the problem sets.

Voorbeeld: normalisatie functie  $e^{-r/a_0}$

$$\int |\psi|^2 d\tau = \int e^{-2r/a_0} d\tau$$

$$= \int_{r=0}^{\infty} r^2 e^{-2r/a_0} dr \cdot \int_{\theta=0}^{\pi} \sin\theta d\theta \cdot \int_{\varphi=0}^{2\pi} d\varphi$$

$$= \frac{1}{4} a_0^3 \cdot 2 \cdot 2\pi$$

$$= \pi a_0^3$$

$d\tau = r^2 \sin\theta dr d\theta d\varphi$

$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$

**Figure 1:** Sample slide from the second lecture, showing, as an example, the normalization of the function  $e^{-r/a_0}$ .

Calculate the following integrals. For more information, read pages 129-131 from the Calculus I lecture notes.

(a)  $\int_0^{2\pi} d\varphi$  ,

(b)  $\int_0^\pi \sin \theta d\theta$  ,

(c)  $\int_{x=0}^1 \int_{y=-1}^1 xy^2 dydx$  .

**Explanation** The primitive of a constant equals the constant times the variable ( $\varphi$  in this case). The primitive of  $\sin \theta$  equals  $-\cos \theta$ . For an integral of more than one variable, you first have to calculate the integral over one variable (e.g.  $x$ ). The other variable (in this case  $y$ ), can be seen as a constant. Next, you calculate the integral over the remaining variable ( $y$ ).

**Figure 2:** Questions from one of the Maple T.A. assignments to practice the calculations needed in the slide in

*Figure 1.* The explanation is shown after the student has submitted the answer. The question is translated from Dutch.

In short, students would prepare for a lecture by taking the warming-up questions, and the mathematics brushing up questions on-line. The teacher could collect their results and comment on them in the lecture. After the lecture, we provided a set of “cooling-down” questions about the lecture. Students could use these questions as a self test, to see whether they had understood the lecture. There were two exams with equal weight: one at mid-term, and one after conclusion of the course.

The results of the introductory diagnostic test were very disappointing for the group as a whole. This made the teacher decide to make the follow-up assignment compulsory. Students were not allowed to take part in the exams without having attempted to take all problem sets. Their grade, however, did not matter.

The advantage of using Maple T.A. is threefold.<sup>vi</sup> Firstly, because of its digital character, students can take the exercises when and where they want. This fits well with our objective to make it the students’ responsibility to take the exercises. Secondly, Maple T.A. makes it possible to set up questions that change somewhat each time a student loads the assignment. This enables the students to take the exercises repeatedly. Finally, students get instant feedback upon completion of the exercise. They thus learn from their previous attempt.

This setup fits well with the work of Edwards<sup>vii</sup>, in which he showed that diagnostic tests in mathematics in combination with follow-up support to those in need can have good results only if test and support combination are carefully monitored.

### Results and Evaluation

To measure the effect of this intervention, the lectures and tutorial sessions were observed. This made it possible to monitor the effect of the Maple T.A. assignments directly and to compare it to the results of the situation in the previous year where no assignments in mathematics were given. The students’ attempts at taking the exercises on-line were also followed, since every attempt was recorded. This way we could see

if students completed the assignments, what questions they found most difficult, and what progress they made. In addition to this, a survey was conducted after completion of the course.

Because the follow-up assignments were made compulsory for all students, as described in section 0, we do not know if our set-up stimulated participation, as was intended. Most students, however, took most assignments more than once. On average students took assignments twice. This shows that they took the brushing up seriously, and indicates a positive effect on participation.

During the lectures we could see that the students were able to better follow the mathematical reasoning of the teacher. To ensure the students were able to comprehend his lecture, the teacher often asked simple questions about the mathematics. For instance, he would ask what the second derivative is of  $e^{kx}$ , with respect to  $x$ . The previous year, students were often not able to answer such simple questions. Now it posed no problems at all.

As mentioned in section 0, the follow-up assignments were made compulsory for all students. Of the 27 students that enrolled for this course, 19 students took part in the mid-term exam and took all corresponding assignments. The final exam was taken by 12 students, who also took the corresponding assignments. Most of the students that dropped out, only took a few of the on-line assignments. In the analysis we leave out these students, because there are several reasons why they dropped out. Some decided to change study. Some had difficulties following all courses in the semester, and made an economical decision not to invest in the Quantum Chemistry course. Furthermore, the students that did not participate in the mid-term exam had an average score of 53% for the introductory test, whereas the students that did participate had an average score of 59%. This shows that the introductory test is not an indicative for dropping out.

The result to the mid-term exam and the average result to the corresponding on-line assignments show a reasonable correlation ( $N=19$ ). One student had an average score of 46%, but got 8.6 out of 10 for the mid-term exam. There were four students who scored higher than 46%, but got a 3 or lower for the mid-term exam. For all the other students it holds that the higher the score to the math tests, the higher their grade to the mid-term exam. This can have two reasons. Firstly these students might have been better motivated, and thus received a better mark for both the math assignments, and the mid-term exam. Secondly, these students might have had benefit from the brushing up.

The results of the final exam shows a different pattern ( $N=12$ ). There does not seem to be a correlation between these results and the average score to the math assignments. This might first of all have been caused by the fact that for this exam proper knowledge of what has been taught in the first half is essential. Secondly, the math assignments did not treat very much additional math, when compared to the assignments in the first half of the course. Some of the assignments were identical to earlier assignments. In other words: for these students it was easy to get a good score for the mathematics, but they had not yet mastered the needed quantum mechanics to successfully take this exam.

Upon conclusion of the course we conducted a survey. In the analysis only the responses of students were included who have at least attended the mid-term exam (12 out of 14 respondents).

Students say they have spent an average of 34 minutes on each Maple T.A. assignment, ranging from 15 to 80 minutes. This is longer than we originally anticipated. From the results of the survey it can furthermore be seen that students rate

their mathematical skills mainly below average; they feel the need to do the remedial exercises. Most of the students said that they benefited from taking the assignments: they could better comprehend the lectures. Also, a majority state that their mathematical skills have improved and that the on-line assignments were a good way to do so. One of the students stated that the assignments were not an effective means and explained additionally that his skills were so poor, that more effort was needed. Some improvement of the technical aspect (e.g. the use of Maple T.A.) is desirable, although none of the students experienced big problems with the tool.

### **Conclusion & Discussion**

We have seen that on-line remedial exercises help students improve their mathematical skills. In doing so, they are better able to follow the lectures and answer elementary questions on the needed mathematics. We also see a positive effect on the results of the mid-term exam: there was a positive correlation between the mid-term exam and the math assignments. However, the absolute grades were rather disappointing: five students passed the mid-term exam, only four passed the final exam. Furthermore, there did not seem to be any correlation between the results to the math assignments and the final exam. This shows that mathematics is essential, but not sufficient to learn quantum chemistry.

We conclude from this that there are other important factors causing students to have difficulties with quantum chemistry. The first factor might still have a mathematical origin. The Quantum Chemistry course requires students to be able think abstractly. However it is not to be expected that doing some additional exercises will in itself result in better abstract thinking amongst our students. On the other, hand quantum chemistry is more than just mathematics. New concepts are introduced that do not follow easily from everyday experience. For students it is difficult to interpret experiments that physicists find so convincing evidence for quantum mechanics (e.g. the double slit experiment for electrons). The relation between such experiments and the concepts of quantum theory is currently a subject for a PhD study by one of the authors.

The results presented here show that the brushing up is useful, but more needs to be done. Some improvements can be made to the follow-up program. More importantly, we need more students to pass for the Calculus I exam. To achieve this, starting in 2006-2007, chemistry students will get three hours tutorial, instead of two. Furthermore, the Calculus course will give special attention to the mathematics involved in the Quantum Chemistry course. This is done by introducing exercises with notation that corresponds to that used in the Quantum Chemistry course.

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