

# **The Challenges Facing Pre-service Education: Addressing the Issue of Mathematics Subject Matter Knowledge among Prospective Primary Teachers**

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Research has indicated that it is not necessary to study mathematics to degree level to teach mathematics effectively at primary level (Greaney et al, 1999; Cooney, 1999). Consensus exists however that graduating teachers do require a deep and rich knowledge of mathematics to a certain level in order to facilitate optimum mathematics teaching and pupils learning (An Roinn Oideachas agus Eolaíochta, 2002).

Internationally dissatisfaction exists regarding the standard of mathematics subject matter knowledge evident among both qualified and prospective primary teachers (Ma, 1999, Farmer et al, 2003). Concern is also evident within the Irish context, particularly within the Colleges of Education (Wall, 2001; An Roinn Oideachas agus Eolaíochta, 2002; Corcoran, 2005). The author (MH) aimed, through a two cycle action research methodology, to identify the existing level of mathematics subject knowledge among prospective teachers in one Irish College of Education in an effort to address weaknesses and develop 'deep' subject knowledge among participants through the development and implementation of a custom-built intervention. After a brief presentation of the characteristics of the research study, this paper will focus on the reconnaissance stage of the first cycle of the action research project.

## **1. Introduction**

A mathematics lecturer responding to the N.C.C.A. (2006: 29) consultation document declared that "...students need their best teachers at a young age, teachers who really know what they are doing and really understand the simplicity of what they are doing. Once confidence is in place at a young age, I think the other issues...will right themselves". One must ask the question: what is required to develop such teachers? As well as more generic knowledge such as general pedagogical knowledge, Schulman (1986) proposed that in order to be able to teach any subject effectively teachers require three categories of subject knowledge: content knowledge, subject specific pedagogical content knowledge and curriculum knowledge. For the purposes of this study, the authors' focus is on 'mathematics content knowledge', which is often referred to as 'mathematics subject matter knowledge' (Rowland et al, 2005). Mathematics subject matter knowledge 'refers to the amount and organisation of knowledge per se in the mind of teachers' (Schulman, 1986: 9). This type of knowledge includes the facts and concepts of a discipline, its organizing frameworks, and the ways in which propositional knowledge has been generated and established (An Roinn Oideachas agus Eolaíochta, 2002). Corcoran (2005,b) refers to subject matter knowledge as 'mathematical literacy'.

## **2. Mathematics Subject Matter Knowledge: A Must!**

It is only "In the past two decades teachers' knowledge of mathematics has become an object of concern" (Hill et al, 2004: 11). Research suggests the principal reason for this was that many nations especially the U.S. and U.K., following dissatisfaction with their pupils' relatively poor mathematical performance in international comparative studies when compared to their Eastern peers, were eager to identify the 'causes' of this unsatisfactory scenario (Wall, 2001). Consensus existed among policy makers that "...no curriculum teaches itself" (Ball et al, 2005: 14) and that improved subject matter knowledge among teachers would facilitate enhanced learning among pupils. Such

thinking resulted in increased status and attention being assigned to the issue of teachers' mathematics subject matter knowledge (Wall, 2001; Goulding, 2003).

Nowadays, it is fair to say that "...there is general agreement that teachers' personal knowledge of mathematical content to be taught is the cornerstone of teaching for proficiency" (American Federation of Teachers (A.F.T.), 2005: 1). Ball et al (2005) suggest that the nature of a teacher's subject matter knowledge affects his/her ability to make apt decisions regarding the most appropriate instructional materials, presentation, emphasis, and sequence of instruction. The UK's Training and Development Agency for Schools (T.D.A.) (2006) asserts that a teacher requires a high level of knowledge and understanding in order to be able to confidently and effectively develop pupils' mathematical knowledge and understanding. This assertion receives support from various studies which report a positive correlation between teachers' mathematics subject matter preparation and their effectiveness in the classroom, which in some cases was measured through pupil achievement (The Interstate New Teacher Assessment and Support Consortium (I.N.T.A.S.C.), 1995; Tirosh et al, 1998; An Roinn Oideachas agus Eolaíochta, 2002; Hodgen, 2003; Ball et al, 2005; Hill et al, 2005).

### **3. How much Mathematics Subject Matter Knowledge is enough?**

While consensus exists that practicing primary teachers require 'deep' and 'rich' mathematics subject matter knowledge, it is essential at this stage to resolve what constitutes 'appropriate' mathematics knowledge for teaching. Ball (1990) challenges the assumptions that mathematical concepts and procedures addressed at primary level are easy. The question remains as to "...exactly what and how much mathematics they need to know and be able to do..." (A.F.T., 2005: 1).

It is now recognised internationally that subject matter knowledge beyond a certain 'threshold' is not associated with greater pupil achievement i.e. primary teachers do not need to study mathematics to degree level (Greaney et al, 1999; Burke, 2000; An Roinn Oideachas agus Eolaíochta, 2002; Goulding, 2003). This finding does not suggest that a teacher's knowledge of mathematics is irrelevant to the quality of mathematics teaching and learning. While in the past, there was a perception from some quarters that "...elementary teachers need very little ..." mathematics subject matter knowledge (Rowland et al, 2005: 256) i.e. that it was sufficient for teachers to be able to do anything required of pupils (a 'minimalist' view), this position is challenged by the argument that teachers require more than 'learner knowledge' given that pupils can ask questions that extend beyond the formal curriculum (Prestage and Perks, 1999; Ball et al, 2005). The minimalist view also assumes that any well-educated adult possesses the subject matter knowledge required to teach at primary level. This reflects the belief that 'He who knows mathematics, knows how to teach it' (Boero et al, 1996). Ball et al (2005) propose that while teachers need to be able to use reliable algorithms i.e. demonstrate 'common' mathematics subject matter knowledge, procedural knowledge alone is insufficient for teaching.

Corcoran (2005 (b)) suggests that a certain kind of mathematics subject matter knowledge is needed to teach the subject effectively at primary level, additional to that required by those pursuing other mathematically intensive careers e.g. accountants (Ball et al, 2005). Hodgen (2003: 104) expresses a similar view referring to "...the need for primary teachers to know mathematics differently". Ball et al (2005) refer to this knowledge as 'specialised'. Hill et al (2005: 373) suggest that the 'specialised'

mathematical knowledge required for the work of teaching is vast given that this ‘work’ includes

explaining terms and concepts to students, interpreting students’ statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematics concepts, algorithms and proofs.

To be able to meet the aforementioned demands, a teacher must possess conceptual understanding of the various mathematical concepts and procedures as well as recognising and understanding the interconnections between them (Kessel and Ma, 2000; Conference Board of the Mathematical Sciences (C.B.M.S.), 2001; Ball et al, 2005). This belief receives support from Schulman (1986) who highlighted that subject matter knowledge required for teaching included “...both facts and concepts in a domain but also why facts and concepts are true and how knowledge is generated and structured in the discipline” (Hill et al, 2005: 376).

On the other hand, ‘weak’ mathematics subject matter knowledge is associated with less competent mathematics teaching (An Roinn Oideachas agus Eolaíochta, 2002). The coping strategies utilised by such teachers include avoiding topics altogether, overdependence on the text, limitation of interaction and a focus on rules and procedures as isolated facts (An Roinn Oideachas agus Eolaíochta, 2002; Barber and Heal, 2003). In such contexts pupils must depend on memorization rather than understanding which in turn leads to the “...failure to lay the groundwork for future development of student understanding” (Leavy and O’ Loughlin, 2006: 54).

#### **4. The ‘Health’ of Elementary Teachers’ Mathematics Subject Matter Knowledge**

##### *4.1 International Findings*

Internationally, the volume of research exploring the nature of both qualified and prospective elementary teachers’ mathematics subject matter knowledge i.e. what they actually do know, has grown in line with increased status given to the issue (Wall, 2001; Thwaites et al, 2005). There is overall consensus supported by the reports that all is not well (Tirosh et al, 1998; Hodgen, 2003; Tsang and Rowland, 2005). Characteristics of elementary teachers who were deemed to have ‘substandard’ mathematics subject matter knowledge (e.g. UK, US, Hong Kong) include a dependence on rule-bound knowledge, shortcomings in both procedural and relational understanding of concepts, and ignorance to connections and gaps in knowledge (Ball, 1990; Ma, 1999; Rowland et al, 2005; Ball et al, 2005). Such dissatisfaction has resulted in the initiation of an accountability movement within the U.S. and U.K, which in turn led to the introduction of standards (e.g. Qualified Teacher Standards (QTS, UK)) which were rigorously tested at various levels (e.g. numeracy skills test for licensing purposes) (Wall, 2001; Rowland et al, 2005). Such standards had direct implications for the pre-service education in the relevant education systems (I.N.T.A.S.C., 1995; Rowland et al, 2005; Ball et al, 2005; Tsang and Rowland, 2005; T.D.A., 2006). In the UK for example, teacher training colleges are required to “...monitor trainee teachers’ progress, give them feedback, review and meet their individual needs, and encourage them to take responsibility for their own development” (T.D.A., 2006: 76).

##### *4.2 The Issue in the Irish Context: A Focus on Prospective Teachers*

It is understandable that one may have the impression that there is little concern regarding the mathematics subject matter knowledge of Irish prospective primary

teachers, given the sparse amount of research on the phenomenon. Wall (2001) and Corcoran (2005) both support this claim stating that of teachers' subject matter knowledge has not been a source of obvious concern in Ireland to date. The fact that the issue has been ignored is reflected within education policy documents. In reality however, the research which has been carried out by a number of individual researchers within the various Colleges of Education (Wall, 2001; Corcoran, 2005; 2005 (b); Oldham, 2005; Leavy and O' Loughlin, 2006) testifies that the mathematics subject matter knowledge that Irish student teachers bring to teacher education is largely unsatisfactory reflecting the characteristics of their international peers. Unlike the UK and US systems, the mathematics subject matter knowledge 'required' by Irish primary teachers is quite limited (Wall, 2001; Corcoran, 2005 (b)). Once prospective teachers achieve the minimum entry requirement i.e. D3 at Ordinary/Higher Level Leaving Certificate Mathematics, they are not obliged to provide any further evidence of their mathematics subject matter knowledge (Corcoran, 2005).

### **4.3 Potential Causes of the Problem**

Ball et al (2005) suggest that the phenomenon of inadequate mathematics subject matter knowledge among prospective elementary teachers internationally should not come as a surprise, given that they are products of the mathematics education systems that are deemed unsatisfactory. Accordingly the C.B.M.S. (2001) suggests that gaps in prospective teachers' mathematical backgrounds are systemic rather than personal failings as their only mathematical experiences equate mathematical strength with computational proficiency.

A similar scenario exists within the Irish system. While this specific issue has not received the publicity warranted, concern regarding substandard mathematical skills evident among Leaving Certificate students generally has been escalating for some time (N.C.C.A., 2005). Consensus is now widespread among students, practitioner and professional groups alike that pre-tertiary mathematics education in its present form is short-changing those who wish to pursue further education (Oldham, 2001; N.C.C.A., 2006). Regardless of level of study or grade achieved, many such students are deemed 'at risk' or under-prepared' on entry to tertiary level courses as they lack basic understanding of concepts in fundamental areas (Murphy, 2002; Lyons et al, 2003; N.C.C.A., 2006). Consensus exists that the nature of predominant classroom practices, especially at the senior cycle of second level education i.e. exam-led, teacher-led didactic approach focusing on rules and procedures which are likely to be examined, is not conducive to the provision of a high quality mathematics education which develops conceptual understanding among students (Murphy, 2002; Lyons et al, 2003; N.C.C.A., 2006; Hourigan and O' Donoghue, 2007). Such a pre-tertiary mathematics experience which prioritises memorisation over connection-making is not conducive to the development of 'deep and rich' mathematics subject matter knowledge among prospective primary teachers (Corcoran, 2005; N.C.C.A., 2005; Hourigan and O' Donoghue, 2007).

The above findings call into question the validity of the minimum mathematics entry requirement provision as a suitable indicator of adequate subject matter knowledge for prospective primary school teachers (See section 4.2). While there have been calls within the Irish context for this requirement to be raised (An Roinn Oideachas agus Eolaíochta, 2002), the authors are not convinced that such adjustments alone are sufficient to address the phenomenon in question given the dissatisfaction which exists

regarding the narrow range of mathematical skills among Leaving Certificate students regardless of the grade (Oldham 2005; N.C.C.A., 2006).

### **5. Motivation for the Study: Response to the Issue**

Into the future, the scenario looks more positive from a pre-tertiary perspective. There is much hope to be gained from the initiation by the N.C.C.A. (2006) of the first root and branch review for almost forty years with a view to making future radical changes to the nature of the post-primary mathematics education (e.g. content, methods and assessment) which would facilitate the provision of a continuum of mathematics learning over the years in formal education e.g. ensure a smooth transition from primary to post-primary and post-primary to tertiary mathematics education (N.C.C.A., 2005; 2006). Undoubtedly for proposals to become a reality, support for teachers at all levels is essential. The authors are hopeful that the envisaged reform will facilitate the ‘tackling of the problem where it arises’ thus facilitating future prospective teachers to enter pre-service education possessing ‘deep’ mathematics subject matter knowledge (N.C.C.A., 2006). In the interim, however, there is no doubt that the nature of the predominant pre-tertiary mathematics experience is exacerbating the demands placed on all tertiary mathematics educators including pre-service educators.

This research study was the author’s (MH) reaction to the phenomenon in question within her working environment, a College of Education. The author’s decision to explicitly address the issue resulted from encountering increasing number of instances of substandard mathematics subject matter knowledge in her daily work with student-teachers (mathematics pedagogy course, teaching practice). In many cases within the teaching practice context, it was clear that poor mathematics subject matter knowledge directly impacted on student-teachers’ perception of or attitude to the subject as well as their ability to transform the knowledge appropriately to facilitate pupil learning. Informal conversations also enlightened the author regarding the effects of substandard subject matter knowledge on student-teachers’ attitudes and feelings (Nitko, 2001). While all of the aforementioned incidences represent mere anecdotal evidence, their repeated nature resulted in the author making a firm commitment to further explore this real but somewhat ‘silent’ issue (Elliot, 1991). The motivation for this particular research study comes from the authors’ deep-seated values in relation to improving the preparedness of prospective primary teachers which in turn will affect the standard of mathematics teaching future generations of pupils will experience. Aware that if the phenomenon was left unchallenged, a vicious cycle of shallow knowledge and negative attitudes was likely to persist, it was considered essential to set in motion a process to address the issue (C.B.M.S., 2001; N.C.C.A., 2006; Hourigan and O’ Donoghue, 2007).

### **6. The Context of this Study**

While Oldham (2005) recommends that the mathematics subject matter knowledge of some entrants to primary teacher education needs enhancement, unfortunately in reality the “...subject matter preparation of teachers is rarely the focus of any phase of teacher education” (Ball, 1990: 465). Despite some changes in recent times, it is still the case for many Irish prospective primary teachers, including those attending the College of Education within this study, that the sole form of preparation for teaching mathematics is the mathematics pedagogy course. As these courses are expected, within limited time constraints, to provide student-teachers with the necessary knowledge to teach mathematics at all primary class levels, it is not surprising that finding the time to explicitly address student-teachers’ mathematics subject matter knowledge proves

problematic (Wall, 2001; An Roinn Oideachas agus Eolaíochta, 2002; Corcoran, 2005; Leavy and O' Loughlin, 2006). Consequently within these courses it is often taken for granted that the mathematical subject matter knowledge relating to the various concepts and procedures was addressed 'somewhere else' e.g. pre-tertiary mathematics. In such contexts no distinction is generally made between knowledge of content and knowledge of how to teach it (Ball, 1990; Wall, 2001; C.B.M.S., 2001; Rowland et al, 2005). Unfortunately in light of the reported nature of many student teachers' pre-tertiary mathematics experience (See section 4.3), such assumptions are unfounded and have serious implications (Oldham, 2005; N.C.C.A., 2006; Hourigan and O' Donoghue, 2007). While the authors initially considered the possibility of extending existing mathematics education courses in order to facilitate the desirable alterations, this option was subsequently deemed unfeasible given students' heavy timetables and workloads. Instead it was decided that some 'extra' provision outside the mainstream provision would be initiated in a bid to meet the identified needs of prospective primary teachers within the College of Education.

The author (MH) endeavoured to develop an initiative to address the phenomenon of substandard mathematics subject matter knowledge, which would reflect the practices of universities worldwide who have set up initiatives aimed at remediation. In line with best practice, the broad objectives for this research would be achieved through a sequence of events moving from 'Diagnosis' to 'Prescription' to 'Aftercare' (Murphy, 2002). This approach would consist of a systematic process of collecting and analysing data regarding the nature of the participating student teachers' mathematics subject matter knowledge which would increase the authors' understanding of phenomenon. Such insight would subsequently facilitate the development of a purpose-built intervention in a bid to provide prospective teachers with an opportunity to develop their mathematics subject matter knowledge of the concepts and procedures they will teach (Murphy, 2002; Edwards, 2003).

### *6.1 The Sample*

While the author was aware that many pre-service providers initiate the process of a diagnosis and remediation programme from entrance to course onwards in order to develop instructional practices based on a sound knowledge base, this was not practical in economic terms in this study (C.B.M.S., 2001; Goulding, 2003; Rowland et al, 2005; Corcoran, 2005 (b)). The sampling technique utilized for the study was purposive i.e. a particular cohort of students within the wider student-teacher population was targeted (Cohen et al, 2000; Mertens, 2005). After discussion with colleagues, the sample selected for the purposes of this study was the cohort of second year prospective teachers. As their mathematics pedagogy course during the spring semester addressed mathematics issues directly related to the cohorts' subsequent senior teaching practice placement, it was suggested that these student-teachers would be optimally motivated to partake in the study as it would be perceived to be relevant to their immediate needs (Rowland et al, 1999). In light of these facts, the authors believed that this particular group of students would prove to be information-rich for the purposes of the present study (Mertens, 2005).

### *6.2 Methodology and Methods*

In line with the pragmatic approach adopted by the authors that no one paradigm ensures a perfect grasp of the 'truth', the methodology and methods were determined on the basis of a 'fitness for purpose' criterion or 'What works?' (Cohen et al, 2000;

Mertens, 2005). Once the author had a clear insight into the objectives of the study, selecting the optimum methodology to ‘fit’ the problem at hand was a straightforward process. As the author (MH) intended to carry out on-the job professional enquiry in order to gain understanding of a problematic situation and subsequently address issues and problems arising within this context, the author selected an action research approach as the most appropriate research methodology (Colleran, 2002; Opie, 2004; Mertens, 2005). The research study falls into two identifiable cycles of diagnosis, intervention implementation and evaluation (i.e. Cycle 1 or Preliminary study and Cycle 2 or Main study). The first cycle began in February 2006 and concluded in May 2006 with a selected cohort of prospective teachers. The lessons from the implementation and evaluation of cycle 1 provided an improved service during cycle 2. The second cycle, which built upon the learning of the preliminary phase, commenced in February 2007 with the subsequent cohort of student teachers.

The author selected Elliot’s (1991) model of action research on the grounds that this model promoted a smooth transition between the stages and cycles. This model facilitated the development of an ‘initial idea’. This statement of what the researcher wished to improve was based on reflection of both the review of relevant research and experiences i.e. to address the issue of substandard mathematics subject matter knowledge among prospective teachers (Cohen et al, 2000; Colleran, 2002). The reconnaissance stage facilitated the quantification of the extent to which substandard subject matter knowledge posed a problem for pre-service teachers within the College of Education in question (Murphy, 2002). The investigation process was further focused through the development of appropriate testable hypotheses and research questions. Subsequently it was necessary to select, develop and administer the data collection methods deemed most compatible with the research questions bearing in mind the study’s context and participants e.g. time available, population size. The findings of the previous stage facilitated the design of an appropriate general plan i.e. the development and implementation of a suitable intervention as well as the evaluation of its effects. Analysis of the evidence from various data collection methods facilitated the researcher in drawing conclusions regarding the effectiveness of the initiative in meeting its proposed goals and in making the necessary modifications to a subsequent cycle (Elliot, 1991).

Throughout both cycles, a multi-method approach was adopted i.e. both qualitative and quantitative strategies were used to address research questions in an effort to secure broader and better results through triangulation (Mertens, 2005). The methods employed serve to add to the researchers’ knowledge of the phenomenon (Cohen et al, 2000). The various stages of the action research and the corresponding data collection methods used are summarized in Appendix 1.

### *6.3 Ethical Considerations*

The study meets the ethical requirements of MIC and University of Limerick Ethics Committees. Prior to and during the implementation of the research study it was necessary to fulfill a number of ethical obligations. Initially it was necessary to formally seek consent and support from management and colleagues within the educational setting to pursue the research and access to the selected sample group. Subsequently in order to achieve the respect and trust of the potential participants within the study, it was essential to ensure that their dignity, privacy and interests were respected at all times. Measures utilized included the presentation of information (both orally and in written

form (information sheet)) to the entire cohort of prospective participants regarding the nature of the initiative (e.g. voluntary participation), its purposes and procedures as well as the potential benefits of participation prior to the initiation of each cycle. Students were advised that they could withdraw from the study without penalty at any stage of the study. While within the preliminary study confidentiality was assured to all participants, the promise of anonymity was viable for participants within the main study as project identification codes were developed for each participant. Throughout the initiative, arrangements were made to provide feedback (in various forms) as a matter of form or on request. Prior to the commencement of the respective cycles of action research participants were requested to give their consent for their personal data to be utilized for initiative and research purposes (Cohen et al, 2000).

## **7. Cycle 1- Reconnaissance Stage: The Process of Determining if Concerns are Warranted?**

The reconnaissance stage of the initial cycle strove to acquire evidence of the actual nature of the mathematics subject matter knowledge among the participating prospective primary teachers for the purposes of providing feedback to the participants themselves as well as informing the development of an intervention to address the issue of inadequate subject knowledge (Nitko, 2001; Murphy, 2002; Corcoran, 2005).

### *7.1 The Data Collection Method*

Given the weaknesses of the pre-tertiary system and consequently the Leaving Certificate mathematics grades as a predictor of 'preparedness', it was necessary to objectively assess student-teachers' existing levels of subject knowledge (Nitko, 2001). The author decided to administer a mathematical pre-test. The purpose of the test was to determine the extent to which student-teachers have an acceptable grasp of the essential mathematical subject matter knowledge required to teach at primary level which in turn would facilitate decision making on a number of levels (See hypotheses/research questions in Appendix 2) (Tsang and Rowland, 2005).

Decisions regarding the testing procedure were made after due consideration of practices and procedures within a variety of third level institutions internationally (Murphy, 2002; Edwards, 2003; Learning and Teaching Support Network (L.T.S.N.), 2003; Corcoran, 2005; Mertens, 2005; N.C.C.A., 2006). As part of this process, the author analysed a number of measurement instruments previously utilized to gauge student teachers mathematics subject matter knowledge e.g. Sigma-T, SKIMA in a bid to evaluate their fitness of purpose within the present study. The author wished that the test instrument would reflect the mathematics subject matter knowledge required to fulfill their professional obligations i.e. reflecting the learning outcomes of Revised mathematics curriculum to the highest level (An Roinn Oideachas agus Eolaíochta, 2002; Nitko, 2001). Because the various instruments were not developed exclusively to reflect the Irish Mathematics Curriculum (1999), various mismatches in both content and thinking skills tested meant that none were deemed fit for this study's requirements (Wall, 2001; Corcoran, 2005). Subsequently the authors felt that the development of a purpose-built test was necessary (Cohen et al, 2000).

#### *7.1.1 Development of Pre-test Instrument*

The author (MH), with the support of a colleague, set about creating an appropriate paper-based assessment instrument tailored to the needs of the study (Nitko, 2001; L.T.S.N., 2003). A criterion-referenced test was selected as the most appropriate tool

within the present study i.e. it facilitated the testing of participants' ability to demonstrate desirable mathematical concepts and procedures (Cohen et al, 2000; Wall, 2001).

In terms of specific topics and knowledge to be measured, given that the Curriculum and its accompanying Teachers Guidelines are the only official documents which specify the mathematics that primary teachers need to know, it was decided that the test items would represent the breadth and depth of the Revised mathematics curriculum i.e. reflect curriculum objectives for each strand (i.e. Number, Shape & Space, Measures, Algebra and Data) and mathematical skills (i.e. Applying & Problem Solving, Integrating and Connecting, Reasoning, Implementing, Understanding and recall) promoted within the curriculum at the highest level (N.C.C.A., 1999; Wall, 2001; Nitko, 2001; Cohen et al, 2000; Corcoran, 2005). The development of appropriate test items was facilitated by referring to curriculum documents, existing measures as well as textbook activities (Wall, 2001; Ball et al, 2005). The subject knowledge required within the instrument did not, for the most part, extend beyond the 6<sup>th</sup> class mathematics curriculum objectives. In total the test instrument consisted of 41 'completion' items, most of which were closed-response tasks requiring participants to construct their own answers (in the rough work section), and independently place the final answer in the space provided (Nitko, 2001). A small minority of the test items were 'response-choice' items requiring participants to explain/justify the response deemed correct (Cohen et al, 2000; Nitko, 2001). The format selected facilitated the author in gaining insight into participants' level of understanding as well as the nature of their misunderstandings (Wall, 2001). The instrument does not, however, demonstrate the traditional characteristics of a criterion-referenced test (i.e. mastery determined by performance on 3 items per objective) (Nitko, 2001). While the development of additional items for the curriculum objectives was possible, such a test would be extremely off-putting for potential participants given the voluntary nature of participation and the unreasonable amount of time required to complete such an instrument (Wall, 2001).

The authors were aware that this mathematics test focuses primarily on 'common' content knowledge or knowledge that many adults would be able to demonstrate i.e. working through computations and solving problems (Goulding, 2002). There were a few exceptions within the tests, where 'specialised' knowledge e.g. item requiring the justification or rejection of the statement '*A square is a special type of rectangle- True or False? Explain*'. The author is not advocating that "If you can "do" these items, you can teach them" (Ball, 1990: 462). Although at face value it could be argued that high performance in the test does not guarantee that participants possess the subject matter knowledge required to teach mathematics, participating student teachers were requested to reflect as to whether they possessed the conceptual knowledge associated with each object i.e. if they could explain why (Hill et al, 2004). Therefore it was intended that the pre-test would act as a 'self-audit' for participants to make them aware of the mathematics concepts and procedures required for the purposes of teaching which in turn would facilitate them in identifying weaknesses in both their 'common' and 'specialised' mathematics subject matter knowledge (Goulding, 2003).

#### *7.1.2 Ensuring the Reliability and Validity*

The 'content validity' of the test i.e. the extent to which the assessment instrument is representative of the mathematics subject knowledge that primary teachers require was assured through the distribution of the draft instrument to a 'jury of experts (i.e.

colleagues and practicing primary teachers of the senior classes requesting them to review the ‘fitness-for- purpose of the instrument and provide feedback (Mertens, 2005). Piloting of the test on a group of student teachers (N=89) facilitated feedback regarding issues such as clarity of directions, ambiguity of wording as well as completion times etc. (Cohen et al, 2000; Wall, 2001; Mertens, 2005).

#### *7.1.3 Data Collection: Administration of Pre-test*

Prior to test administration, the author ensured that all members of the cohort were made aware of the initiative generally and the assessment process (See section 6.3). During the information session details provided included the purpose and content of the test instrument as well as scoring criteria and subsequent processes (e.g. feedback, use of results) (Nitko, 2001; L.T.S.N., 2003). While it was possible to provide student-teachers with reading lists to facilitate preparation, the author wished the results to reflect what mathematics subject knowledge the students had at their fingertips (Cohen et al, 2000; Wall, 2001; Murphy, 2002).

The test was administered on the second week of semester in a bid to ensure that the test results reflected the subject knowledge achieved through pre-tertiary mathematics experiences. The test was administered in independent timeslots i.e. outside the students’ mathematics pedagogy sessions, which meant that participants had to sacrifice some of their personal time in order to partake. The test was initially administered in large venues on Wednesday, February 15<sup>th</sup> (Week 2) at 1p.m. and 3p.m.). Although ‘on paper’ all student-teachers within the cohort were in a position to attend at least one of the two times selected, it soon became apparent that this was not the case. In order to facilitate all interested students to take the test, two extra testing sessions were subsequently organised for Week 3 (Tuesday, February 21<sup>st</sup> at 2p.m. and Thursday, February 23<sup>rd</sup> at 10a.m.). Students were informed of this opportunity via both announcement and notice.

The same conditions i.e. instructions regarding test completion and time allocated were provided for each testing session to ensure the reliability of findings. Prior to taking the pre-test, participants were required to complete a ‘consent form’ stating that they had received adequate information and that they permitted the author to obtain and use their personal information (e.g. test results, Leaving Certificate grade) for the purposes of the initiative and research (Cohen et al, 2000; Wall, 2001). They were also requested to indicate contact information for feedback purposes. In all, 163 undergraduate student-teachers attended and completed the pre-test i.e. approximately one third of the total cohort of second year students invited to participate in the initiative.

#### *7.1.4 Test Correction, Data analysis and Use of Findings*

Reflecting common practice within the numerate sector, the author selected an ‘all or nothing’ marking scheme i.e. correct answer = 1 mark; incorrect answer= 0 marks, partially completed item = 0 marks (Cohen et al, 2000; Murphy, 2002). The quantitative data available as a result of the pre-test process ranged from nominal (correct/incorrect) to ordinal (LC grades, level of mathematical study) to ratio data (total scores, scores in subsections). Data analysis carried out on the relevant data, using the tools of SPSS Version 14, included descriptive statistical measures as well as inferential statistics. Further analysis of the collected data was facilitated through the comparison of subgroups within the student population. The author sought insight into the apparent relationship between student teachers’ level of mathematics study to date and their

performance within the pre-test (Wall, 2001; Murphy, 2002). While the author also acquired additional qualitative information into the nature of student-teachers deficits i.e. through error analysis, the nature and outcomes of this process will not be addressed explicitly here (Wall, 2001; Nitko, 2001).

The data analysis processes, both quantitative and qualitative, facilitated the author to gain insight into the needs of the participants (areas of strength and weakness, nature of misconceptions), which would prove invaluable in informing and guiding decisions regarding the nature of the optimum support structures (e.g. content, process and instructional decisions) (Wall, 2001; Murphy, 2002; Nitko, 2001; L.T.S.N., 2003). Participants also received individualised feedback regarding their performance in the pre-test within three weeks of sitting the diagnostic test i.e. on Monday, March 6<sup>th</sup> (Week 5) by e-mail (or phone if so indicated) in the form of raw scores for each test item (1/0), and as the number of correct answers as a function of the total responses in each section (strand) (e.g. number score/16) and the whole test (score /41) (Cohen et al, 2000). The author chose not to provide participants with a specific 'cut-off point' to guide their decisions regarding intervention participation, given the limited nature of this single indicator of their mathematics subject matter knowledge. While the preliminary test feedback was numerical, due to the pressures of large numbers and time, each participant was invited to view their script thus facilitating them to self-assess their strengths, misunderstandings and needs. Unfortunately few participants availed of this service. It was not viable to 'return' the tests as the author planned to administer the same instrument (with perhaps minor modifications) to subsequent cohorts in order to develop a data base of student performance. The feedback mechanism facilitated participants to evaluate their existing level of mathematical readiness and subsequently make informed decisions (L.T.S.N., 2003).

## *7.2 Findings*

### *7.2.1 Profiling the Populations' Strengths and Weaknesses*

The initial profiling of 'strengths and 'weaknesses' made the author as well as the participants' themselves aware of levels of understanding within various concepts tested (Elliot, 1991; Wall, 2001). The average score for the test was 26.46 out of a possible 41 or 65% correct. While the modal score was 31, participants' scores ranged from 6 to 39. It is interesting that no student achieved full marks in the test. The test scores were categorized for the purposes of analysis i.e. 0-21; 21-30 and over 31. The author found that while almost one fifth (17.8%) of the population achieved a score of 20 or less (i.e. the lowest band), 33.1% of the population performed at the top band (i.e. 30-39). Almost half of the pre-test respondents performed in the middle band i.e. 49.1%.

Subsequently participants' performance within the various subsections was analysed in order to gain further into the strengths and weaknesses of the participant population. The average score for the 'Number' section, the Measures section and Algebra section were 63%, 61% and 61% respectively, while the average scores for the other two test subsections namely 'Shape and Space' and 'Data' were 67% and 76% respectively. While informative, no attempt was made to compare the populations' performance within the subsections, given the possibility of variability in difficulty between the subsections. Further information was required to facilitate the analysis of populations' specific strengths and weaknesses (Wall, 2001).

The population's performance on individual test items was also gauged. Percentages of mastery suggested that a number of individual items posed little difficulty for the participants, demonstrating that these students possessed adequate 'common knowledge' of the concepts and procedures in question. For example while almost all members of the population (over 90%) successfully completed items requiring them to add and subtract decimals and find the Highest Common Factor of two numbers, a majority of them (approximately 75-90%) demonstrated an understanding of place value, implemented the procedures to calculate the perimeter and mean, utilised knowledge of the angles within a circle/triangle to label specific angles, and solved word problems within real world contexts requiring an understanding of ratios, percentages, value for money, time, and directed numbers. The author was acutely aware, however, that a high level of mastery on specific items was not an assurance of conceptual knowledge among the population.

An informal analysis of students' rough work during correction suggested that basic fact errors or 'slips' or an inability to recall facts and definitions e.g. meaning of 'mode' or prime number or formulas and rules e.g. the angle with a circle  $=360^\circ$  were the source of many 'errors'. Also a number of items on which a high proportion of the population demonstrated inadequate 'common' subject matter knowledge were an immediate source of concern in light of the importance of these fundamental concepts within the curriculum and that conceptual knowledge of these concepts and procedures would have facilitated item solution. For example while only 63.8% successfully completed an item requiring the subtraction of mixed fractions (requiring decomposition), just over half (51.5%) of the population successfully found the area of a shaded shape. Further items caused substantial difficulty for many participants, with fewer than half of the participants demonstrating the 'common' subject matter knowledge required to solve them. While 41.1% of participants successfully found the 'product' of two decimal numbers, 46% demonstrated the ability to divide by a decimal. An item requiring students to demonstrate an understanding of the connections within mathematics i.e. to order a list of fractions, decimals and percentages proved too demanding for 57.7% of the population (i.e. 42.3% correct). A further source of unease resulted from the fact that a number of items requiring higher order skills such as connection making and reasoning proved problematic for all except two fifths of the population e.g. finding the area of an irregular shape (44.2% correct), solving problems involving percentage profit (39.9%) and speed (38.7).

The small numbers of participating student-teachers demonstrating 'common' subject matter knowledge on items requesting the knowledge and understanding of concepts and procedures such as the Lowest common multiple (31.3%), division involving fractions (27.6%) and fractions within a story problem context (22.1%) is a reasonable source of concern in light of the demands of the Revised Curriculum (1999) (N.C.C.A., 1999). While in many cases these students may have forgotten the 'algorithm' or the 'rule', their inability to work through the solution suggests that they lack an understanding of the concepts required to make the concept associations and connections required. Overall the results demonstrate that while participants demonstrate high levels of proficiency in items which can be solved through the utilisation of recall and procedural knowledge in relatively context free or simple contexts, many participants demonstrated limited conceptual understanding when completing items requiring connection making and/or reasoning. In light of these findings it is questionable as to whether such students possess 'specialised' subject

knowledge. Support for this belief comes from the fact that the test item which explicitly assessed the participants' 'specialised' mathematics subject matter knowledge i.e. the properties of 2-d shapes was poorly answered, with only 24.7% of the participants' able to explain the relationship.

### 7.2.2 Level of Mathematics Experience as a predictor of 'Common' Mathematics Knowledge

The authors were also interested to investigate the characteristics of the participants using the pre-test i.e. their previous mathematics experience in a bid to explore the ability of such factors to predict participants' 'common' mathematics subject matter knowledge as gauged by the pre-test. Of the 159 student-teachers whose Leaving grade could be reliably verified, 66% studied the Leaving Certificate mathematics course at Ordinary level (OL), while the remaining 34% had taken Higher Level (HL). Further insight into the Leaving Certificate performance of the pre-test population i.e. % of population who achieved various grades can be attained from the table below.

**Table 1:** Participants' Leaving Cert Grades

Grade	Ordinary Level	Higher Level
A (A1/A2)	30.8%	3.1%
B (B1/B2/B3)	27%	17%
C (C1/C2/C3)	6.9%	9.4%
D (D1/D2/D3)	1.3%	4.4%

While the fact that the entire spectrum of Leaving Certificate grades is represented in the group who participated in the pre-test may mean that many of these student-teachers are conscientious and wish to take the opportunity to gain insight into present level of subject matter knowledge. This response may also suggest that these student-teachers, regardless of their achievements in mathematics, lack confidence in their personal knowledge and understanding of mathematical concepts and procedures required for teaching the subject. Analysis of the relationship between Leaving Certificate and pre-test performance, as measured by Spearman's rank order co-efficient, uncovered that there was a strong positive and significant correlation [ $r = .657$ ,  $n = 159$ ,  $p < 0.01$ ] between the two factors. Cross-tabulation of these factors provided some interesting findings. Having taught this course the author was not surprised to find that the two OL 'D' students achieved scores of 15 and 21 (out of 41). The poor performance in this pre-test among students who have achieved grades far exceeding the minimum requirements, however, was a source of disquiet. Approximately half of the OL 'C' students (i.e. 5 of the 11 students) and 23% of the OL 'B' students (10 of the 43 students) achieved a score of less than 20. Among student-teachers who achieved an 'A' in the OL ( $N = 49$ ), 6% performed at a similar level. These findings further question the reliability of the Leaving Certificate grades as a valid predictor of student-teachers' ability to demonstrate mathematical concepts and procedures required to present the Revised Primary Curriculum (See section 4.3).

Exploration of the relationship between the populations' level of mathematics study and their pre-test scores provides insight into previous proposals that studying mathematics to degree level is not essential for primary teachers (See Section 3). While there was a significant positive correlation between the two factors [Spearman's rank order correlation:  $r = .447$ ,  $n = 163$ ,  $r < 0.01$ ], analysis, through cross-tabulation, found that while higher proportions of the prospective teachers studying mathematics beyond

Leaving Certificate demonstrated high levels of ‘common’ subject matter knowledge mastery, as gauged by the pre-test, equivalent subject knowledge was demonstrated by a number of participants who had not studied mathematics beyond Leaving Certificate. Therefore, while 81.3% of ‘major’ students (studying mathematics to degree level- 13 students) achieved a score exceeding 30, 63.2% of ‘minor students (studied mathematics as an academic subject in 1<sup>st</sup> year of their preservice course- 12 students) and 23% of ‘Leaving Certificate’ students (i.e. 20 students) demonstrated similar levels of proficiency.

## **8. Conclusion: Overall Contribution of the Pre-test within the Initiative**

The reconnaissance process provided insight into the nature of the phenomenon of mathematics subject matter knowledge within one Irish College of Education. The authors believe that the high response rate suggests that many of the participants had existing personal concerns regarding their mathematics subject matter knowledge.

While this test does not begin to reflect the demands placed on teachers’ mathematical subject matter knowledge when responding to pupils answers/queries, many prospective teachers are demonstrating substantial difficulties. Pre-test findings highlight a significant mismatch between the fundamental ‘common’ subject matter knowledge required to teach at primary level and that demonstrated by a proportion of the population within the pre-test. The above levels of mastery reflect previous findings both nationally and internationally that a proportion of prospective teachers demonstrate thin knowledge, generally relying on rules and procedures as opposed to conceptual understanding of mathematical concepts and procedures (Ball, 1990; Ma, 1999; Wall, 2001; Corcoran, 2005; Rowland et al, 2005; Ball et al, 2005). It is no surprise in light of the reported nature of prospective teachers’ pre-tertiary mathematics experiences (See section 4.3) that a substantial proportion of the population despite having satisfied minimum entry requirements demonstrated weak ‘common’ subject knowledge of a number of the concepts and procedures required for teaching.

The author felt that despite the limitations of the pre-test instrument used, the reconnaissance stage of the initial cycle of action research within this study fulfilled a number of extremely important functions. As well as providing valuable information on the characteristics and needs of the population, which facilitates informed decisions regarding the nature of optimum support structures, the pre-test acted as a device to “surface and challenge” prospective teachers’ awareness of and desire to develop existing levels of subject matter knowledge (Goulding, 2002: 2).

## **Appendices:**

### **Appendix 1: Stages of Action Research Cycles and Methods Used**

#### **Cycle 1:**

- **Identify Initial Idea:** Address the issue of substandard mathematics subject matter knowledge among prospective primary teachers
- **Reconnaissance:** Collection of data regarding the phenomenon (*Mathematical pre-testing*)
- **Devise general plan:** Develop needs-led intervention
- **Implement action step:** Pilot implementation of the intervention programme i.e. Professional mathematics Programme

○ **Monitor implementation and effects:** Monitor and evaluate implementation stage and effects (*Usage Statistics, Reflective Diary, Mathematical post-test, Survey*)

### **Cycle 2:**

○ **Reconnaissance:** Explain any difficulties, failures and effects through pilot case report and collect additional data regarding the phenomenon from the new cohort (*Mathematical pre-testing, Pre-survey*)

○ **Amended plan:** Amend plan on the basis of pilot discoveries

○ **Implement next action step:** Implement the main stage of the intervention i.e. Professional mathematics programme

○ **Monitor implementation and effects:** Monitor main stage of intervention implementation and effects, explaining difficulties, developments and findings through case study report, making suggestions for future cycles of the process (*Usage Statistics, Reflective Diary, Mathematical post-test, Post-Survey, Interview, Observation of Teaching*)

### **Appendix 2: Cycle 1: Reconnaissance Stage Hypotheses and Research Questions**

○ Pre-tertiary experiences do not provide student teachers participants with the mathematics subject knowledge deemed essential to teach mathematics effectively (as indicated by the pre-test)

○ The entry requirement in mathematics within the Leaving Certificate for entry to teaching (O/H D3) does not guarantee that student teachers possess adequate mathematics subject knowledge

○ There is a relationship between student teachers' mathematics Leaving Certificate performance (level and grade) and their existing levels of mathematics subject knowledge as indicated by the pre-test

○ There is a relationship between student teachers' level of mathematics study and their existing levels of mathematics subject knowledge as indicated by the pre-test

○ What types of mathematical gaps and misconceptions were evident among participants?

○ Were there any particular content areas and/or mathematical skills where participants demonstrated substantial difficulties?

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