# Introducing the history of mathematics to third level students with weak mathematical backgrounds: a case study

**Ciarán Mac an Bhaird**

*Department of Mathematics, National University of Ireland, Maynooth*

## Abstract

**Many students who traditionally struggle with basic aspects of Mathematics have little or no concept of Mathematics as a living and growing subject area. They appear not to appreciate the background of the day-to-day Mathematics that they study, and which some of them may one day teach. These students generally have no exposure to this material and are unlikely to investigate the History of Mathematics independently. In this paper we will discuss the incidents that lead us to consider how to introduce students with weak mathematical backgrounds to the History of Mathematics. We will briefly mention the reasons why there are such significant numbers of students with these issues. We will also provide some detail on the first steps taken by the Mathematics Support Centre in the National University of Ireland Maynooth to try to introduce the background and context of Mathematics to these students. Based on the feedback we have received to date, we will discuss if these initiatives have had a positive impact in terms of students' attitudes and results.**

## 1. Origins of this approach

The idea of introducing students with weak mathematical skills to the History of Mathematics arose as a result of some particular incidents in the Mathematics Support Centre (MSC) at the National University of Ireland Maynooth (NUIM). On one occasion, while discussing aspects of Calculus, a student asked if '*…Calculus was just made up to torture students?*' On another occasion, while addressing issues with zero and infinity, a student asked '*Why are there all these crazy rules?*' It is a common occurrence during weaker students' initial visits that they think they are '*really stupid or slow*'. They get frustrated because they are struggling with basic concepts, whilst in the middle of a higher level Mathematics course.

As we started to explain the historical background to these mathematical topics, we discovered that these students had rarely thought of this before. Subsequently these students have little or no concept of Mathematics as a continuous and growing subject. They see little of its development and few recognize the connections between the different areas of Mathematics. They generally have no idea why they are studying their course material. They try to apply the rules and hope to get close enough to the answer. This lack of understanding is well documented; see [1], [2] and [3].

We find that explaining that some mathematical concepts have taken thousands of years to develop into their current form actually reassures the students. They see the bigger picture and realize that mathematical understanding is not automatic, it involves success and failure. These are an essential part of the process of understanding. Students who realize this can overcome the 'fear' of tackling new mathematical material.

As a result of these observations, we wanted to regularly expose students with weak mathematical backgrounds to the origins of the relevant mathematical material.

## 2. Why is this approach needed?

There is widespread concern about the numbers of students who have basic mathematical problems. Recent reports, [2] and [3], contain detailed analysis of these issues in the teaching and learning of Mathematics at second level in Ireland. Some of the main factors listed include: bad publicity for Mathematics, negative attitudes towards the subject, little understanding of the context or background of Mathematics, rote learning. [2], [4] and [5] have highlighted possible impacts of these problems. Low attainment in Mathematics is cited as a contributing factor in low enrolment and retention rates in science and technology courses, [6] and [7]. There is also significant international research on these issues [8].

Introducing new teaching methods to address these issues should be a priority. Using the History of Mathematics is one approach that should help with basic understanding of the material. We believe that this can help alter the students' image of Mathematics and show that there is much more to Mathematics than memorization.

## 3. Existing resources and similar ideas

### 3.1 History of Mathematics Material

There is extensive material available on the History of Mathematics, for example [9], [10] and [11]. There are also a number of websites containing extensive information, for example [12]. However, we believe that students with weak mathematical backgrounds are unlikely to use these resources independently. At the very least, they will not use these resources until they are made aware of the role that the History of Mathematics can play in their mathematical education. It is safe to assume that a student struggling with fractions will find a text on the History of Mathematics very daunting.

### 3.2 History of Mathematics Courses

Many universities offer the History of Mathematics as a standard undergraduate course. However, in general these courses are available for specialist Mathematics students only. For example, in NUIM, the History of Mathematics is provided to final year Mathematical Studies students only [13]. These are the students who are most likely to become Mathematics teachers. They can introduce and use some of this material when they are teachers of Mathematics, thus tackling the problem from the start.

In NUIM, it is first year students who have the most common basic problems with Mathematics; 41% of the visits to the MSC in 2007-2008 were from first years. These students get little or no exposure to the background and origins of Mathematics. Only those continuing on into third year Arts will see the material at all. This is too late for the weaker students, so it is important that we try to introduce the material earlier by other means.

### 3.3 Using the History of Mathematics in Mathematics Education

The role of the History of Mathematics in Mathematics Education is not under scrutiny. Extensive material is available which discusses this issue, see [14], [15] and [16]. [15] contains several interesting papers, for example Grugetti [pp. 29-35] states that '*An historical… analysis allows teachers to understand why a certain concept is difficult for the student…*' and '*In observing the historical evolution of a concept, pupils will find that mathematics is not fixed and definitive*'. Student teachers who are being trained to use the History of Mathematics in their teaching are interviewed by Isaacs, Mohan Ram and Richards [pp. 123-133]. One student commented '*I have never been taught maths in this way before, rather I have always been given a set of problems and been told to solve them, which has made maths a boring subject. Yet, when faced with a question to answer, and the history behind a particular thing, it makes further questions easier to handle and I found it stays ingrained in the memory better.*'

These are issues we would like to address. However, almost without exception, the articles and material are geared towards specialist mathematical students. This gives us valuable insight into the process of introducing the History of Mathematics to students in general. However, how do we address this issue with the weaker students?

## 4. First steps towards implementation

We want to introduce the History of Mathematics to weaker students, students who are unlikely to attend History of Mathematics courses, read the textbooks or be involved in similar initiatives. Simply pointing the students towards the available resources would not have any significant impact. If they are already having difficulties with Mathematics, why would they research the background of the material that is already confusing them?

We decided to develop resources which would supplement the normal mathematical material available.

### 4.1 Mature Students Mathematics Refresher Course

#### 4.1.1 Implementation

In 2007 the author was asked to design a Mathematics Refresher Course for incoming Foundation students to NUIM. These students are typically mature students who have not completed the Leaving Certificate (A Level). The five day course is run prior to the start of the academic year. Students then proceed to a Foundation Certificate Course in Science, Economics or Engineering. When they complete the Certificate, they are qualified to enter into the first year of an equivalent degree.

The students are mathematically very weak so the topics covered include: counting, addition, subtraction, multiplication, division, zero, fractions, decimals, percentages, algebra, functions etc. The author designed the course from a historical point of view. Each of the topics are discussed and developed from their origins, and then explained with examples. The topics also overlap to reflect the natural development of Mathematics.

#### 4.1.2 Feedback

In 2007 and 2008, 40 Foundation Course students completed basic evaluation forms which were distributed by the Mature Students' Office. They were asked to grade their answers from 1 to 10, where 1 indicated poor and 10 indicated excellent.

- Question one gauged the students' evaluation of course content. The average mark was 9.18.
- Question two gauged the students' evaluation of course presentation. The average mark was 9.40.
- Question three gauged the students' evaluation of how well the course met their expectations. The average mark was 9.25.

One student commented '*I've done Maths at secondary school and I learned more at this foundation week of Maths*'.

The Mature Student Officer stated that a greater percentage of students exposed to this Refresher Course completed the Foundation Certificate Course and continued on to degree level. The Mature Students Office is introducing a Mathematics Refresher Course for mature students entering degrees in 2009-2010. They have requested that the author design a similar course aimed to a slightly higher level. The Adult Education Office in NUIM provides 'Outreach' Certificate Programmes in Finance and Economics. They have requested permission to use the Mathematics Refresher Course.

The feedback and subsequent decisions to use the course in other projects is very positive. A complete evaluation process is underway to determine the best methods to update the course to the students' benefit and how to include more mathematical topics.

### 4.2 Mathematics Support Centre (MSC)

The MSC was set up in NUIM in October 2007. A complete analysis of the services the MSC provides and the impact it has had is available [17]. The MSC aims to help students who are struggling with any aspect of Mathematics. A significant number of these students struggle with basic concepts.

#### 4.2.1 Implementation

One of the principal objectives of the MSC is to try to use context and background to explain basic mathematical concepts. Due to the huge numbers who attend the MSC, it has been difficult to implement an exact program to date. However, the tutors do their utmost to maintain this objective at all stages.

#### 4.2.2 Feedback

Though there is no conclusive evidence on the use of the History of Mathematics in the MSC, there is evidence to suggest the MSC is having impact on students' grades and their attitudes towards Mathematics. In particular, the MSC appears to have a significant impact on the weaker students, see [17, pp. 30-31]. This is encouraging, as these are the students who we target with our approach.

Students who attended the MSC quoted the ability of tutors to give context and background to the problems they were having, as a major source of influence and motivation.

As the MSC develops, we hope to implement a more exact program for our approach. Consequently, we can conduct a more precise investigation into the students' attitudes towards the use of context and history in the drop-in sessions.

### 4.3 Secondment with Coventry University

The author was offered a secondment opportunity by **sigma** (Centre for Excellence in Mathematics and Statistics Support) in 2008. This offer was based on his interest in using the History of Mathematics and it was agreed that he could work on methods to introduce it to weaker students.

#### 4.3.1 Implementation

The secondment involved a week of collaboration with Dr. John Goodband, Coventry University. We decided that the most effective approach was to provide a single double sided A4 page to supplement certain sections of the Engineering Mathematics First Aid Kit (EMFAK) [18]. The EMFAK is a comprehensive resource, it is widely available and used in most Mathematics Support Centres.

The author worked specifically on the History of Mathematics. Material on Complex Numbers and Linear Algebra was produced. The content of these pages includes an initial summary of the origins of that area of Mathematics. It also addresses common misconceptions and contains a description of the progression of the topic. Finally there is a timeline of major developments in the subject area until modern times. The descriptions are brief and designed for students who would otherwise know little about this material. The pages are available in the Mathematics Support Centres in Loughborough, Coventry and Maynooth. They are attached to the related pages in the EMFAK. They will also be available on-line in the near future.

### 4.3.2 Feedback

The pages were completed in September 2008, so there has been little opportunity to gauge exact student reactions. However, initial comments from staff and students have been very positive. The co-ordinator of the on-line Mathematics Proficiency Course in NUIM has requested additional history pages on other topics. Funds are also available from **sigma** to continue with this project and the author aims to complete a comprehensive set to accompany the EMFAK.

These reactions are encouraging and we will have a clear indication of the appeal of these handouts at the end of the 2008-2009.

## 5. Conclusions

As a result of our experiences in the MSC, we have started these three connected projects to help promote the use of the History of Mathematics when helping students with mathematical weaknesses. The projects are relatively new, so there is little statistically significant data to date. However, we believe we have sufficient evidence to argue that using the History of Mathematics can have a significant and lasting influence on weaker students' attitudes and grades. This has wider repercussions and should impact Mathematics poor public image.

We hope to continue with these projects and provide comprehensive programs to introduce this material to the weaker students. Continuing this work will also give us more data, which will help determine the exact impact of this approach.

## References

1. Hourigan, M. and O'Donoghue, J. (2007) Mathematical under-preparedness: the influence of the pre-tertiary mathematics experience on students' ability to make a successful transition to tertiary level mathematics courses in Ireland. *International Journal of Mathematical Education in Science and Technology*, Vol. 38 (No. 4):461-476.

2. Lyons, M., et al. (2003) *Inside Classrooms: a Study of Teaching and Learning*. Dublin: Institute of Public Administration.

3. National Council for Curriculum and Assessment (NCCA) (2005), *Review of Mathematics in post-primary education (a discussion paper)*. Dublin, NCCA.

4. Cosgrove, J., Shiel G., Oldham, E. and Sofroniou, N. (2004) A survey of Mathematics teachers in Ireland. *The Irish Journal of Education*, Vol. 35:20-44.

5. Curriculum and Examinations Board (1986) *Mathematics Education: Primary and Junior Cycle Post-Primary. Curriculum and Examinations Board Discussion Paper*, Dublin.

6. Organisation for Economic Co-operation and Development (1999), *Measuring student knowledge and skills: A new framework for assessment*. Paris, OECD.

7. Irish Department of Education and Science (2002), *Task Force on the Physical Sciences, Report and Recommendations*. Dublin, Government Stationary Office.

8. Picker, S.H. and Berry, J.S. (2001) Investigating pupils' images of mathematics. *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4:49-56.

9. Boyer, C. (1991) *A History of Mathematics*. New York, John Wiley & Sons, 2nd Ed.

10. Eves, H. (1990) *An Introduction to the History of Mathematics*. Philadelphia, Saunders College Pub., 6th Ed.

11. Fauvel, J. and Gray, J. (1987) *The History of Mathematics*. Basingstoke, MacMillan Education in association with the Open University.

12. MacTutor History of Mathematics archive, University of St. Andrews, Scotland. Accessed via http://www-history.mcs.st-and.ac.uk/ (16 June 2009).

13. History of Mathematics, MT382A, National University of Ireland Maynooth, accessed via http://www.maths.nuim.ie/undergrad/ugmoduledescriptions.shtml #Level300 (16 June 2009).

14. Furinghetti, F. (2000) The history of mathematics as a coupling link between secondary and university teaching. *International Journal of Mathematical Education in Science and Technology*. Vol. 31 (No. 1):43-51.

15. Katz, V.J. (2000) *Using history to teach Mathematics. An international perspective*. USA, MAA .

16. Pengelley, D.J. (2003) "A graduate course on the role of history in teaching mathematics." In Bekken O. et al, eds. *Study the Masters: The Abel-Fauvel Conference*.

17. Mac an Bhaird, C., O'Shea, A. and Morgan, T. (2008) *Mathematics Support Centre Annual Report National University of Ireland, Maynooth*. Department of Mathematics, NUIM.

18. Engineering Maths First Aid Kit, mathcentre, Pearson Education Ltd. Accessed via http://www.mathcentre.ac.uk/staff.php/packs/firstaidkit (16 June 2009).